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HEAT AND MASS TRANSFER EFFECTS ON FLOW PAST PARABOLIC STARTING MOTION OF ISOTHERMAL VERTICAL PLATE IN THE PRESENCE OF FIRST ORDER CHEMICAL REACTION

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ABSTRACT

An exact solution of unsteady flow past a parabolic starting motion of the infinite isothermal vertical plate with uniform mass diffusion, in the presence of a homogeneous chemical reaction of the first order, has been studied. The plate temperature and the concentration level near the plate are raised uniformly. The dimensionless governing equations are solved using the Laplace transform technique. The effect of velocity profiles are studied for different physical parameters, such as chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number, and time. It is observed that velocity increases with increasing values of thermal Grashof number or mass Grashof number. The trend is reversed with respect to the chemical reaction parameter.

Keywords: Parabolic; homogeneous; chemical reaction; first order; isothermal; vertical plate.

INTRODUCTION

Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single-phase volume reaction. In well-mixed systems, the reaction is heterogeneous if it occurs at an interface and homogeneous if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of the first order if the rate of reaction is directly proportional to the concentration itself. Chambre and Young (1958) have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das, Deka, and Soundalgekar (1994) have studied the effect of a homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Mass transfer effects on a moving isothermal vertical plate in the presence of a chemical reaction has been studied (Das et al., 1999), and the dimensionless governing equations were solved by the usual Laplace transform technique.

Natural convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat has been studied by Gupta, Pop, and Soundalgekar (1979) using a perturbation method. Kafousias and Raptis (1981) extended this problem to include mass transfer effects subjected to variable suction or injection. Soundalgekar (1982) studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh (1983). Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Kumar (1984). The skin friction for an accelerated vertical plate has been studied analytically by Hossain and Shayo (1986). Mass transfer effects on an exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion was studied by Jha, Prasad, and Rai (1991) and Muthucumaraswamy, Dhanasekar, and Easwara Prasad (2012). Agrawal, Samria, and Gupta (1998) studied free convection due to thermal and mass diffusion in a laminar flow of an accelerated infinite vertical plate in the presence of a magnetic field. Agrawal et al. (1999) extended the problem of unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate with a transverse magnetic plate. The governing equations are tackled using the Laplace transform technique. It is proposed to study the effects of flow past an infinite isothermal vertical plate subjected to parabolic motion with uniform mass diffusion in the presence of a chemical reaction of the first order. The dimensionless governing equations are solved using the Laplace transform technique. The solutions are in terms of exponential and complementary error function.

MATHEMATICAL ANALYSIS

The unsteady flow of a viscous incompressible fluid past an infinite isothermal vertical plate with uniform diffusion, in the presence of a chemical reaction of the first order, has been considered. The x'-axis is taken along the plate in the vertically upward direction and the *y*-axis is taken normal to the plate. At time $t' \le 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞ . At time $t' > 0$, the plate is started with a velocity $u = u_0.t'^2$ in its own plane against the gravitational field, the temperature from the plate is raised to T_w, and the concentration level near the plate is also to C'_w. A chemically reactive species, which transforms according to a simple reaction involving the concentration, is emitted from the plate and diffuses into the fluid. The reaction is assumed to take place entirely within the stream. Then, the usual Boussinesq approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - k_l (C' - C'_\infty) \tag{3}$$

With the following initial and boundary conditions:

$$u = 0, \qquad T = T_\infty, \quad C' = C'_\infty \qquad \text{for all } y, t' \le 0$$

$$t' > 0: \; u = u_0 . t'^2, \;\; T = T_w, \;\; C' = C'_w \quad \text{at} \quad y = 0 \tag{4}$$

$$u \to 0 \qquad T \to T_\infty, \qquad C' \to C'_\infty \quad \text{as} \qquad y \to \infty$$

On introducing the following non-dimensional quantities:

$$U = u\left(\frac{u_0}{\nu^2}\right)^{1/3}, \;\; t = \left(\frac{u_0^{\,2}}{\nu}\right)^{1/3} t', \quad Y = y\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \; C = \frac{C' - C'_\infty}{C'_w - C'_\infty}$$

$$Gr = \frac{g\beta(T - T_\infty)}{(\nu . u_0)^{1/3}}, \; Gc = \frac{g\beta(C' - C'_\infty)}{(\nu . u_0)^{1/3}}, \; K = K_l\left(\frac{\nu}{u_0^{\,2}}\right)^{\frac{1}{3}}, \; \Pr = \frac{\mu C_p}{k}, \;\; Sc = \frac{\nu}{D} \tag{5}$$

Eqs. (1) to (3) reduce to the following dimensionless form:

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \tag{8}$$

The corresponding initial and boundary conditions in dimensionless form are as follows:

$$\begin{aligned} & U = 0, \qquad \theta = 0, \qquad C = 0 \quad \text{for all } Y, t \le 0 \\ t > 0: \quad & U = t^2, \qquad \theta = 1, \qquad C = 1 \quad \text{at} \quad Y = 0 \\ & U \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty \end{aligned} \tag{9}$$

The dimensionless governing Eqs. (6) to (8) and the corresponding initial and boundary conditions (9) are tackled using the Laplace transform technique.

$$\theta = erfc(\eta\sqrt{\Pr}) \tag{10}$$

$$C = \frac{1}{2}\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \tag{11}$$

$$U = \frac{t^2}{3}\left[(3 + 12\eta^2 + 4\eta^4)\; erfc(\eta) - \frac{\eta}{\sqrt{\pi}}(10 + 4\eta^2)\exp(-\eta^2)\right]$$

$$+dt\left((1+2\eta^2 Pr)erfc(\eta\sqrt{Pr})-\frac{2\eta\sqrt{Pr}}{\sqrt{\pi}}\exp(-\eta^2 Pr)-(1+2\eta^2)\,erfc(\eta)\right.$$
$$\left.+\frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right)$$
$$+e\left(erfc(\eta)-\frac{\exp(at)}{2}[\exp(2\eta\sqrt{at})\,erfc(\eta+\sqrt{at})+\exp(-2\eta\sqrt{at})\,erfc(\eta-\sqrt{at})]-\frac{1}{2}[\exp(2\eta\sqrt{ScKt})\,erfc(\eta\sqrt{Sc}+\sqrt{Kt})+\exp(-2\eta\sqrt{ScKt})\,erfc(\eta\sqrt{Sc}-\sqrt{Kt})]+\right.$$
$$\frac{\exp(at)}{2}\left[\exp\left(2\eta\sqrt{Sc(K+a)t}\right)erfc\left(\eta\sqrt{Sc}+\sqrt{(K+a)t}\right)+\right.$$
$$\left.\left.\exp\left(-2\eta\sqrt{Sc(K+a)t}\right)erfc\left(\eta\sqrt{Sc}-\sqrt{(K+a)t}\right)\right]\right)$$

(12)

where, $a=\frac{K\,Sc}{1-Sc}$, $d=\frac{Gr}{1-Pr}$, $e=\frac{Gc}{a(1-Sc)}$ and $\eta=\frac{y}{2\sqrt{t}}$

RESULTS AND DISCUSSION

For physical understanding of the problem, numerical computations are carried out for the different physical parameters *Gr*, *Gc*, *Sc*, and *t* depending on the nature of the flow and transport. The value of the Schmidt number (*Sc*) is taken to be 0.6, which corresponds to water vapor. Furthermore, the values of the Prandtl number (*Pr*) are chosen such that they represent air (*Pr* = 0.71). The numerical values of the velocity are computed for different physical parameters, such as the chemical reaction parameter, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number, and time. Figure 1 illustrates the effects of the concentration profiles for different values of the chemical reaction parameter ($K = 0.2,2,5,10$) at $t = 0.4$. The effect of the chemical reaction parameter is important in the concentration field (Muthucumaraswamy & Valliammal, 2010; Muthucumaraswamy & Radhakrishnan, 2012). The profiles have the common feature that the concentration decreases in a monotonic fashion from the surface to a zero value far away in the free stream. It is observed that the concentration increases with decreasing chemical reaction parameter. The velocity profiles for different values of the chemical reaction parameter $(K = 0.2,2,5), Gr = 5 = Gc, \mathrm{Pr} = 0.71$ and $t = 0.2$ are shown in Figure 2. It is observed that the velocity increases with decreasing chemical reaction parameter. Figure 3 demonstrates the effects of a different thermal Grashof number (*Gr* = 2, 5), mass Grashof number (*Gc* = 5, 10), *K* = 2 and *Pr* = 0.71 on the velocity at *t* = 0.2. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number. The velocity profiles for different values of time $(t = 0.3,0.4,0.6,0.8), K = 2, Gr = 5$ and $Gc = 5$ are presented in Figure 4. The trend shows that the velocity increases with increasing values of time. The effect of the velocity profiles for different values of the Schmidt number $(Sc = 0.16,0.3,0.6), Gr = 5 = Gc, \mathrm{Pr} = 0.71$ and $t = 0.2$ are shown in Figure 5. It is observed that the velocity increases with decreasing values of the Schmidt number.

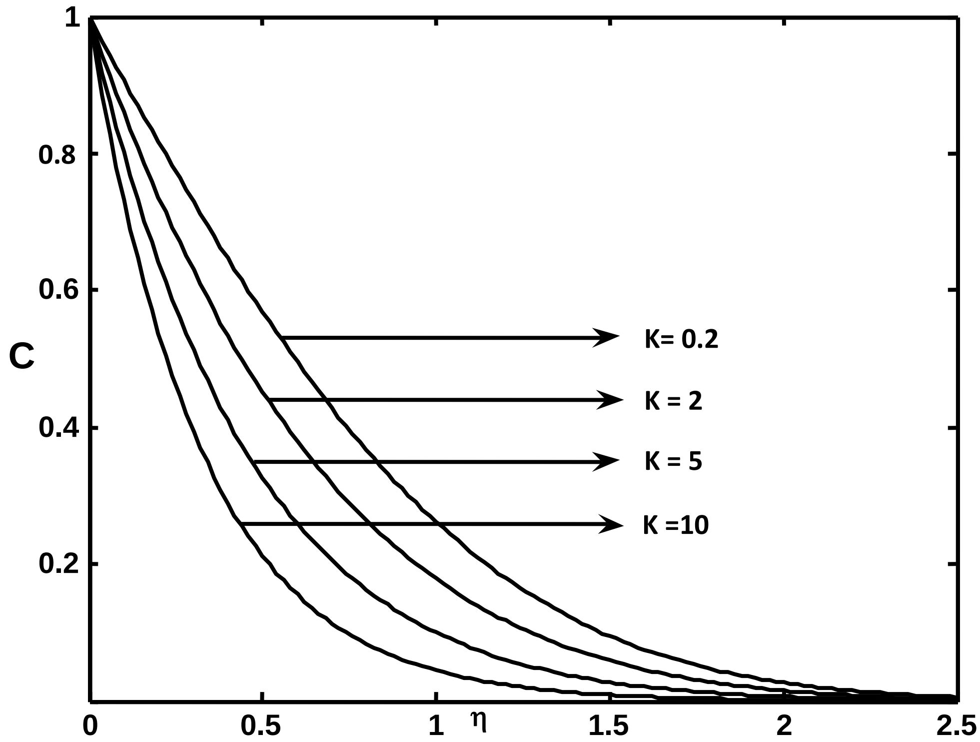

Figure 1. Concentration profiles for different values of K.

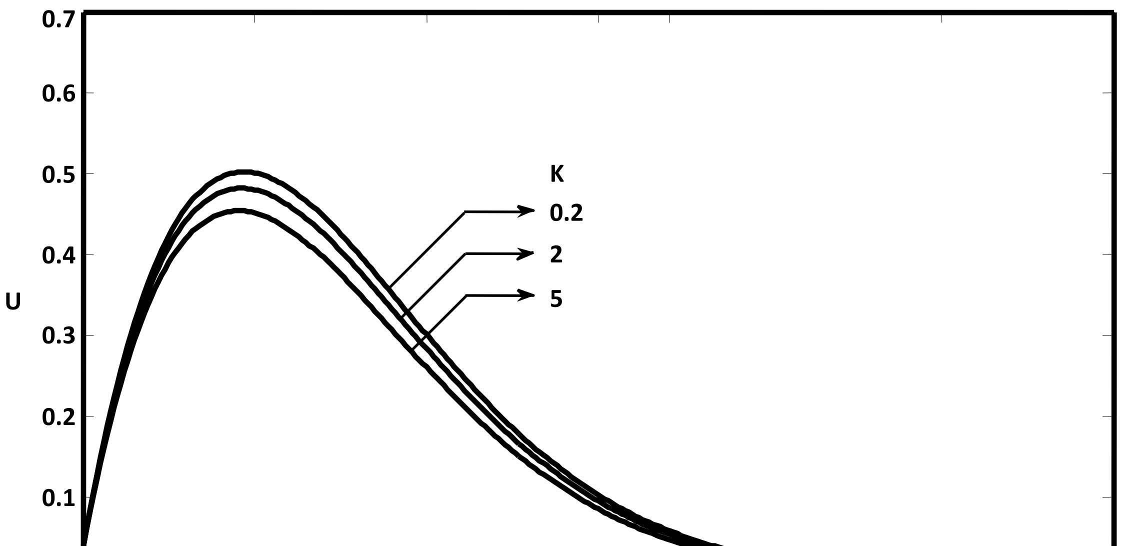

Figure 2. Velocity profiles for different K.

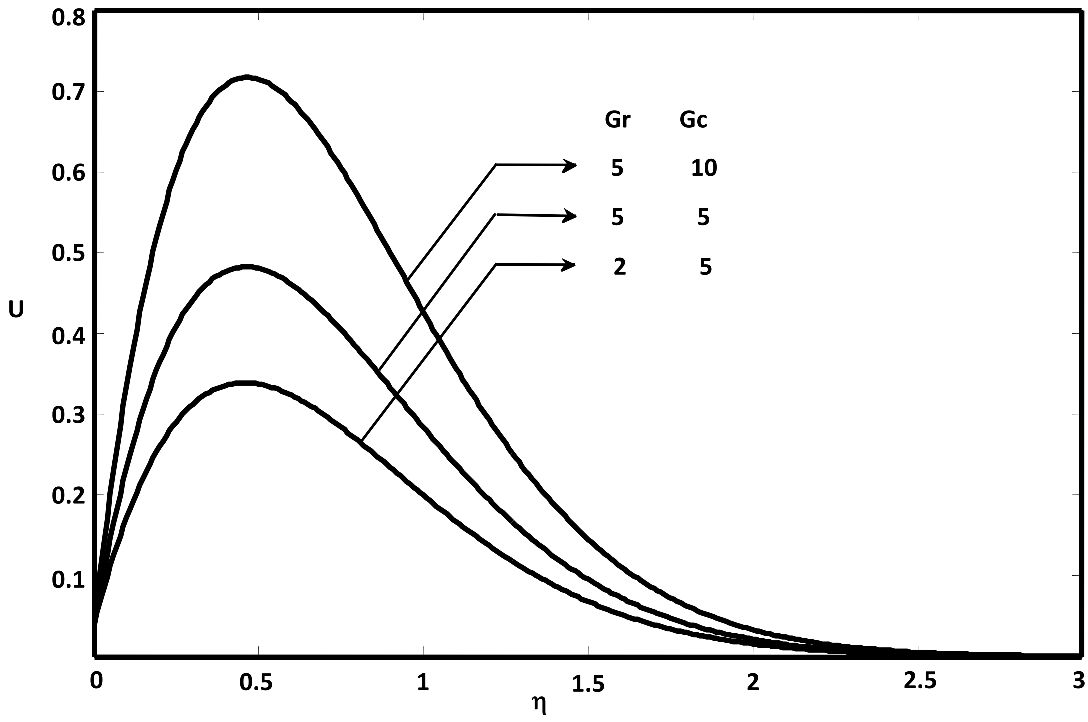

Figure 3. Velocity profiles for different Gr & Gc.

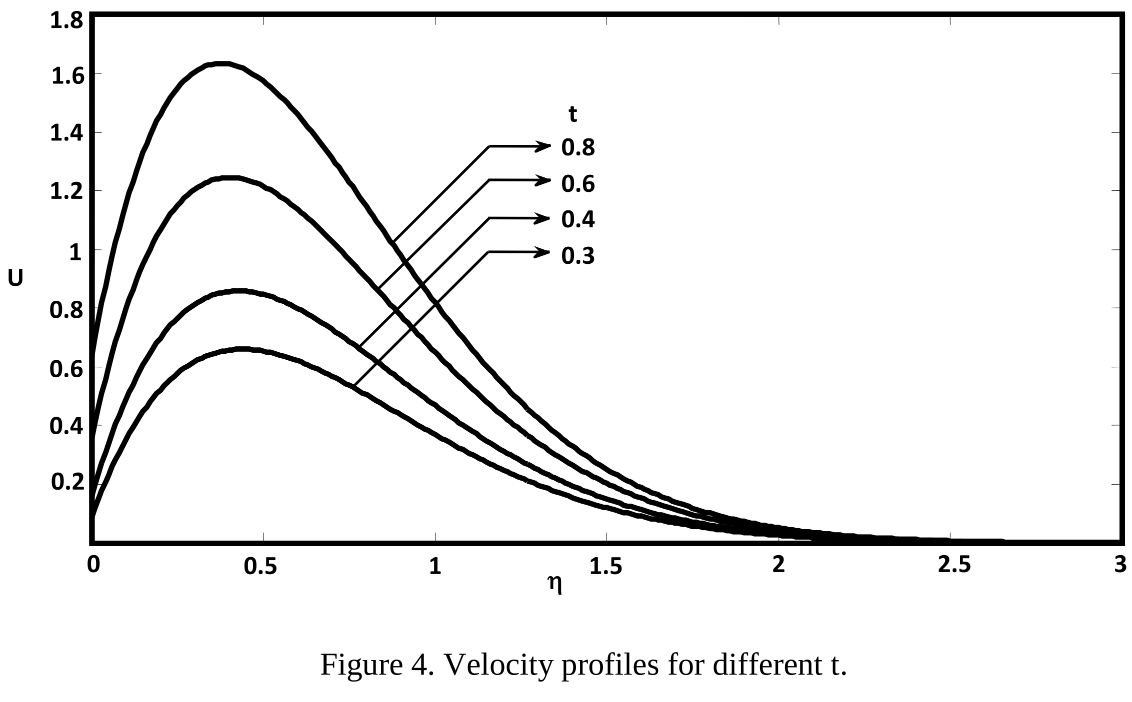

Figure 4. Velocity profiles for different t.

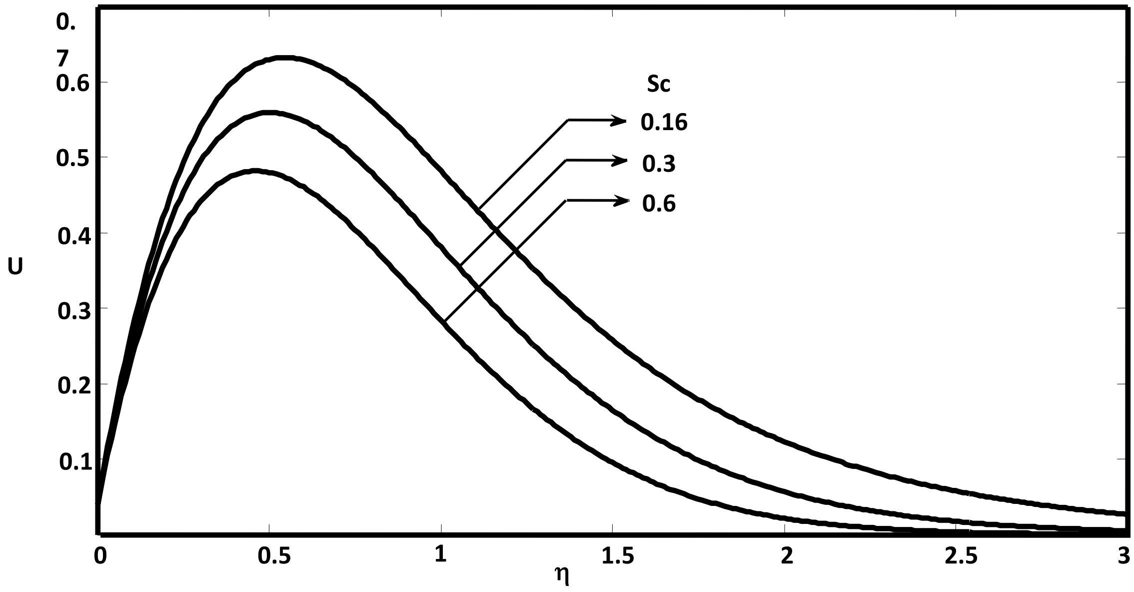

Figure 5. Velocity profiles for different Sc.

CONCLUSION

An exact solution of flow past a parabolic starting motion of the infinite isothermal vertical plate with uniform mass diffusion, in the presence of chemical reaction of the first order, has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of the temperature, the concentration, and the velocity fields for different physical parameters, such as chemical reaction parameter, thermal Grashof number, and mass Grashof number are studied graphically. The conclusions of the study are as follows:

(i) The velocity increases with increasing thermal Grashof number or mass Grashof number, but the trend is just reversed with respect to the chemical reaction parameter.
(ii) The temperature of the plate increases with decreasing values of the Prandtl number.
(iii) The plate concentration increases with decreasing values of the chemical reaction parameter.

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NOMENCLATURE

A Constants

C' species concentration in the fluid $kg\,m^{-3}$

C dimensionless concentration

C_p specific heat at constant pressure $J.kg^{-1}.k$

D mass diffusion coefficient $m^2.s^{-1}$

Gc mass Grashof number

Gr thermal Grashof number

g acceleration due to gravity $m.s^{-2}$

k thermal conductivity $W.m^{-1}.K^{-1}$

Pr Prandtl number

Sc Schmidt number
T temperature of the fluid near the plate K
t' time s
u velocity of the fluid in the x'-direction $m.s^{-1}$
u_0 velocity of the plate $m.s^{-1}$
u dimensionless velocity
y coordinate axis normal to the plate m
Y dimensionless coordinate axis normal to the plate

Greek symbols
β volumetric coefficient of thermal expansion K^{-1}
β^* volumetric coefficient of expansion with concentration K^{-1}
μ coefficient of viscosity $Ra.s$
ν kinematic viscosity $m^2.s^{-1}$
ρ density of the fluid $kg.m^{-3}$
τ dimensionless skin friction $kg.m^{-1}.s^2$
θ dimensionless temperature
η similarity parameter
$erfc$ complementary error function

Subscripts
w conditions at the wall
∞ free stream conditions