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EFFECT OF SLIP VELOCITY ON MAGNETIC FLUID LUBRICATION OF ROUGH POROUS RAYLEIGH STEP BEARING

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ABSTRACT

This article aims to analyze the performance of a magnetic-fluid-based porous rough step bearing considering slip velocity. The Neuringer-Rosensweig model governs the fluid flow while the velocity slip is modeled by the method of Beavers and Joseph. The bearing surfaces are assumed transversely rough and the transverse surface roughness of the bearing surfaces is characterized by a stochastic random variable with non-zero mean, variance, and skewness. With the usual assumptions of hydrodynamic lubrication, the related stochastically averaged Reynolds' equation for the fluid pressure is solved with appropriate boundary conditions, which is then used to calculate the load-carrying capacity. It is found that although the bearing suffers owing to transverse surface roughness, the performance of the bearing system can be improved to some extent by the positive effect of magnetization, considering the slip parameter at the minimum; at least in the case of negatively skewed roughness. A comparison of this paper with some established investigations indicates that here, the reduction of load-carrying capacity due to porosity and slip velocity is comparatively less, especially, when negative variance occurs. In augmenting the performance of the bearing system, the step ratio plays a central role, even if the slip parameter is at the minimum.

Keywords**:** Rayleigh step bearing; magnetic fluid; roughness; slip velocity; porosity; load-carrying capacity.

INTRODUCTION

The infinite step bearing (also known as the Rayleigh step bearing) is of paramount importance to the theory of hydrodynamic lubrication. In practical applications, such as journal and thrust bearings, the bearing is finite and some side leakage occurs. However, the infinite step is conveniently analyzed because it incorporates all other basic features of a step bearing. The Rayleigh step has been established as the bearing with the highest load-carrying capacity amongst all other possible bearing geometries. Rayleigh (1918) used the calculus of variations to find the optimum film shape for the slider bearing, which can support the highest load. Rayleigh introduced film geometry, in which a step divides the film into two levels of film thickness. Porous bearings do not need an external supply of lubricant during their operation; therefore, their structures are simple and reduce costs. For this reason, the study of porous bearings has attracted the attention of investigators. Bujurke, Naduvinamani, and Jagadeeswar (1990) investigated theoretically the performance of a porous Rayleigh step bearing lubricated by a second-order fluid. It was shown that the maximum dimensionless load-carrying capacity occurred at a slightly smaller step ratio compared with the solid case. The method adopted by Hideki (2005) allowed thermohydrodynamic analysis of the bearings with discontinuous clearance on the sliding surface. Guojun, Chengwei, and Ping (2007) studied the hydrodynamic load support generated by a slip wedge of a slider bearing. The surface slip property was optimized to obtain the maximum hydrodynamic load support. It was observed that the hydrodynamic effect of the slip wedge appeared greater than the traditional geometrical convergent wedge. Rahmani, Shivani, and Shivani (2009) discussed the performance of the Rayleigh step bearing, including the effect of pressure at the boundaries on the optimum parameters. Here, it was observed that the optimum bearing parameters depended strictly on the variations of pressure at the boundaries. Laurent, Pascal, and Rene (2012) presented a geometric and a kinematic model for the Rayleigh step bearing, introducing dimensional parameters, which study the bearing's influence on the bearing design characteristics. The influence of a step on the load-carrying capacity and on the mass flow rate was described by dealing with the model for an optimum geometry in favor of the design application.

All the above studies dealt with conventional lubricants. The use of magnetic fluid as a lubricant in a bearing system has attained considerable importance because of its contribution towards industrial applications. With many specific and particular physical and chemical properties, magnetic fluids have been used widely in bearing sealing, lubrication, grinding, separation, inkjet printing, damper support, and surgical cancer treatment. Magnetic fluids are stable colloidal systems with single-domain magnetic nanoparticles suspended in carrier liquids. The magnetic particles, usually Fe_3O_4 or ε- Fe_3N, are up to 10–20 nm (Huang & Wang, 2008). According to theoretical calculation, magnetized magnetic fluid is capable of supporting loads between parallel plates (Shah, 2003). Therefore, an optimized design of the surface magnetic fields can exploit the advantages of magnetic fluid lubrication. Furthermore, the viscosity of the magnetic fluid will be increased with the external magnetic field strength, which may affect the lubrication properties.

Agrawal (1986) considered the configuration of Prakash and Vij (1973) with a magnetic fluid lubricant and found its performance better than one with conventional lubricant. Bhat and Deheri (1991) extended the analysis of Agrawal (1986) by considering a magnetic-fluid-based porous composite slider bearing with its slider consisting of an inclined pad and a flat pad. Here, it was concluded that the magnetic fluid lubrication increased the load-carrying capacity, did not affect friction, and shifted the center of pressure towards the inlet. Naduvinamani and Siddangouda, 2007a) analyzed a porous Rayleigh step bearing lubricated with a couple stress fluid. It was seen that the couple stress lubricant provided an increase in the load-carrying capacity and decreased the coefficient of friction. Singh and Ahmad (2011) considered a porous inclined slider bearing lubricated with magnetic fluid taking into account the thermal effects with slip velocity. It was shown that the porous matrix accelerated the mean temperature, whereas the slip velocity decelerated it. Here, the effects of slip and permeability factors were not significant with regard to load-carrying capacity. Kashinath (2012) analyzed theoretically the couple stress effect on the squeeze film performance between parallel stepped plates. Here, it was shown that the influence of couple stresses enhanced the load-carrying capacity and decreased the response time compared with the classical Newtonian lubricant. Furthermore, by increasing the step height, the load-carrying capacity decreased. After receiving some run-in and wear, bearing surfaces develop roughness. Sometimes contamination by lubricants and chemical degradation of the surfaces contribute to this roughness. The roughness appears to be random in character. Surface roughness is an important factor when dealing with issues such as friction, lubrication, and wear. It also has a major impact on applications involving thermal or electrical resistance, fluid dynamics, noise and vibration control, dimensional tolerance, and abrasive processes, among others. The theoretical analysis of rough surfaces in lubrication dates back to when Hamilton, Wallowit, and Allen (1966) developed a theory of hydrodynamic lubrication between two parallel surfaces with surface roughness on one or both of the surfaces. Tzeng and Saibel (1967) utilized the stochastic approach to study the effect of one-dimensional transverse surface roughness on a slider bearing. It was concluded that the load-carrying capacity and frictional force increased considerably when surface roughness was taken into account. Later, Christensen and Tonder (1970) extended the model of Tzeng and Saibel (1967) for the case of longitudinal roughness. Patir and Cheng (1978) developed a method to determine the effect of surface roughness on partially lubricated contacts. This method was more versatile than the earlier stochastic theories, because it could be applied readily to any three-dimensional surface roughness structure. Several investigators have employed Christensen & Tonder's approach for analyzing the effects of surface roughness (Ting, 1975; Prakash & Tiwari, 1983; Prajapati, 1991; Guha, 1993; Andharia, Gupta, & Deheri, 1997, 1999; Naduvinamani & Siddangouda, 2007b).

Deheri, Andharia, and Patel (2005) studied a transverse rough slider bearing with a squeeze film formed by a magnetic fluid. This article confirmed that magnetization has a considerable positive effect on overall performance. Deheri, Patel, and Patel (2006) launched an investigation into the behavior of a magnetic-fluid-based squeeze film between infinitely long transversely rough rectangular plates. Naduvinamani & Siddangouda (2007c) presented a theoretical study of the effect of surface roughness on the hydrodynamic lubrication of a porous step slider bearing. The bearing registered an adverse effect for the positively skewed roughness and for all values of standard deviation. In the present article, it has been proposed to study and analyze the performance of a magnetic-fluid-based rough porous Rayleigh step bearing with consideration of the slip velocity.

ANALYSIS

The geometry and configuration under consideration is as shown in Figure 1, where film thickness h is constant over two regions. It is defined as

$$h = \begin{cases} h_1 & 0 \le x \le B_1 \\ h_2 & B_1 \le x \le B \end{cases}$$

In 1964, Neuringer-Rosensweig proposed a simple model to describe the steady flow of magnetic fluids in the presence of slowly changing external magnetic fields. The model consists of the following equations:

$$\rho(\bar{q}.\nabla)\bar{q} = -\nabla p + \eta\nabla^2\bar{q} + \mu_0(\overline{M}.\nabla)\overline{H} \tag{1}$$

$$\nabla.\bar{q} = 0 \tag{2}$$

$$\nabla \times \overline{H} = 0 \tag{3}$$

$$\overline{M} = \bar{\mu}\overline{H} \tag{4}$$

$$\nabla.(\overline{H} + \overline{M}) = 0 \tag{5}$$

Using Eqs. (3)–(4), Eq. (1) becomes

$$\rho(\bar{q}.\nabla)\bar{q} = -\nabla\left(P - \frac{\mu_0\bar{\mu}H^2}{2}\right) + \eta\nabla^2\bar{q} \tag{6}$$

This proves that an extra pressure term $\frac{1}{2}\mu_0\bar{\mu}H^2$ is introduced into the Navier-Stokes equations when magnetic fluid is used as a lubricant.

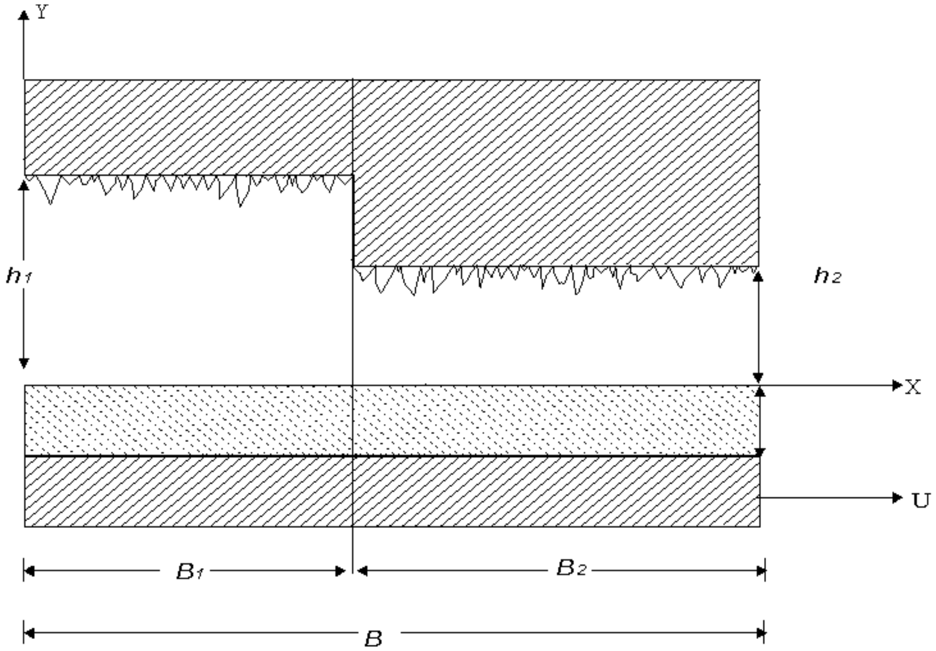

Figure 1. Configuration of the bearing system.

The bearing surfaces are assumed transversely rough. Following Christensen & Tonder (1970), the thickness $h(x)$ of the lubricant film is considered to be

$$h(x) = \bar{h}(x) + h_s \tag{7}$$

Here, $\bar{h}$ is the mean film thickness and h_s is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. h_s is assumed stochastic in nature and governed by the probability density function $F(h_s)$, which is defined by

$$F(h_s) = \begin{cases} \frac{32}{35b}\left(1 - \frac{h_s^2}{b^2}\right)^3 & -b \le h_s \le b \\ 0 & , \quad \text{other wise} \end{cases} \tag{8}$$

where b is the maximum deviation from the mean film thickness. The mean α, standard deviation σ, and the parameter ε, which is the measure of symmetry of the random variable h_s, are defined by the following relationships:

$$\alpha = E(h_s); \qquad \sigma^2 = E[(h_s - \alpha)^2]; \qquad \varepsilon = E[(h_s - \alpha)^3] \tag{9}$$

where E denotes the expected value defined by

$$E\,(R) = \int_{-b}^{b} RF(h_s)dh_s \tag{10}$$

The assumptions of usual hydrodynamic lubrication theory are taken into consideration in the development of the analysis. Following the method adopted in Bhat (2003) and by resorting to the Beavers & Joseph (1967) slip model as well as the roughness model of Andharia et al. (1997, 1999), one arrives at the associated Reynolds-type equation

$$\frac{d}{dx}\left(P - \frac{\mu_0\bar{\mu}H^2}{2}\right) = 6\eta U\frac{S_1 h - h_m}{a(h)} \tag{11}$$

where

$$S_1 = \frac{1+sh}{2+sh}$$

and

$$a(h) = \; h^3 + 3\alpha h^2 + 3(\sigma^2+\alpha^2)h + \varepsilon + 3\sigma^2\alpha + \alpha^3 + 12\varphi H,$$

which is just $h^3 + 12\varphi H$ in the case of smooth bearings.

The following dimensionless quantities are introduced:

$$x^* = \frac{x}{B}\,, H_1 = \frac{h}{h_1}\,, H_m = \frac{h_m}{h_1}, \beta = \frac{B_1}{B}, h^* = \frac{h}{h_1}, B_0 = \frac{B}{h_1}, n = \frac{h_2}{h_1}, {x_1}^* = \frac{x_1}{B}$$
$$\varepsilon^* = \frac{\varepsilon}{{h_1}^3}, \sigma^* = \frac{\sigma}{h_1}, \alpha^* = \frac{\alpha}{h_1}, P^* = \frac{{h_1}^3}{\eta U B^2}P, \mu^* = \frac{\mu_0\bar{\mu}k{h_1}^3}{\eta U}, s^* = sh_1, A(h_1) = \frac{a(h)}{{h_1}^3} \tag{12}$$

Integrating Eq. (11), one can obtain

$$P - \frac{\mu_0\bar{\mu}H^2}{2} \quad = 6\eta U\frac{S_1 h - h_m}{a(h)}\; x + C_1 \tag{13}$$

For entry region, the concerned boundary conditions are:

$$P = 0\,, \qquad \text{at } x = 0$$
$$P = P_c\,, \qquad \text{at } x = \frac{B_1}{B}$$

while the magnitude of the magnetic field is taken from Bhat (2003) as

$$H^2 = kx(x - B_1)$$

where k is the permeability of the magnetic media with value $k = 10^{14}A^2m^{-4}$, which is chosen in order to have a magnetic field of strength over 10^5.

The use of boundary conditions in Eq. (13) leads to

$$C_1 = 0,\quad P_c = \frac{\mu^*}{2}x^*(x^*-\beta) + \frac{6}{B_0}\frac{s^* - H_m}{A(1)}\frac{B_1}{B} \qquad \text{for the entry region} \tag{14}$$

For the second region, by shifting the coordinates to the leading edge, one can find that

$$x_1 = B - x \Longrightarrow {x_1}^* = 1 - x^* \text{ for the region } B_1 \le x \le B$$

where

$$B = B_1 + B_{2,}, 0 \le x_1 \le B_2$$

Also, it is easy to see that

$$\frac{dP}{d{x_1}^*} = -\frac{dP}{dx^*}$$

Here, the associated boundary conditions are:

$$P = 0\,, \qquad \text{at } x_1 = 0$$
$$P = P_c\,, \qquad \text{at } x_1 = \frac{B_2}{B}$$

and the magnitude of the magnetic field is adopted from Bhat (2003) as

$$H^2 = kx(x - B_2)$$

Now, the use of boundary conditions in Eq.(13) offers that

$$C_1 = 0, \quad P_c = \frac{\mu^*}{2} x^*\big(x^* - (1-\beta)\big) + \frac{6}{B_0}\frac{H_m - ns^*}{A(n)}\frac{B_2}{B} \tag{15}$$

where P_c is the common non-dimensional pressure at the step.

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Consequently, the non-dimensional pressure distribution is presented by the equation

$$P^* = \frac{\mu^*}{2} x^*\big(x^* - (1-\beta)\big) + \frac{6}{B_0}\frac{H_m - ns^*}{A(n)}(1 - x^*), \qquad \frac{B_1}{B} \le x^* \le 1 \tag{16}$$

Now, the dimensionless load carrying capacity of the bearing can be obtained from

$$W^* = \int_0^\beta P^* dx^* + \int_\beta^1 P^* dx^* \tag{17}$$

$$= \frac{\mu^*}{12}(4\beta^3 - \beta^2 - 3\beta + 1) + \frac{3}{B_0}\left[\left(\frac{s^* - H_m}{A(1)}\right)\beta^2 + \left(\frac{H_m - ns^*}{A(n)}\right)(1-\beta)^2\right] \tag{18}$$

RESULTS AND DISCUSSION

It can be seen clearly that Eq. (16) determines the dimensionless pressure distribution, while the distribution of load-carrying capacity in non-dimensional form is given by Eq. (18). From Eqs. (16) and (18), it can be seen that the non-dimensional pressure increases by

$$\frac{\mu^*}{2}x^*\left(x^* - (1-\beta)\right) \tag{19}$$

while the dimensionless load-carrying capacity is enhanced by

$$\frac{\mu^*}{12}(4\beta^3 - \beta^2 - 3\beta + 1) \tag{20}$$

compared with the conventional-fluid-based bearing system. It also becomes clear that as the expression for load is linear with respect to the magnetization parameter, the load-carrying capacity increases with the increase of the magnetization parameter

By setting the roughness parameters to zero one gets the behavior of performance characteristics of a magnetic-fluid-based porous step bearing with slip velocity. Furthermore, by considering a non-porous medium and no velocity slip, this study reduces to the performance of a magnetic-fluid-based step bearing. Finally, by taking the magnetization as zero, this investigation turns to the discussion carried out by Cameron (1972) and Basu, Sengupta, and Ahuja (2009). The increasing value of the magnetization parameter results in increased load-carrying capacity, which is visible in Figure 2. It can be seen that at the initial stage, the adverse effects of standard deviation and *n* are relatively less. The effect of standard deviation on the distribution of load-carrying capacity is displayed in Figure 3, which shows clearly that the load-carrying capacity decreases considerably due to the standard deviation. However, this decrease is relatively less in the case of porosity, which can be seen from Figure 3(c). Figure 4 presents the significant effect of skewness on the variation of load-carrying capacity. It is noticed that the positively skewed roughness decreases the load-carrying capacity, whereas the load-carrying capacity increases due to the negatively skewed roughness. Figure 5 suggests that the trends of the variance are similar to that of the skewness. Therefore, the combined effect of variance (-ve) and negatively skewed roughness is significantly positive. The fact that porosity decreases the load-carrying capacity considerably can be seen from Figure 6. However, for large values of *n*, the decrease in load-carrying capacity is the more significant. Finally, the slip velocity and film thickness ratio are also responsible for reducing the load-carrying capacity, which can be determined from Figures 7–9. Thus, for improving the performance of a bearing system, the slip parameter deserves to be kept at a minimum.
A close look at some of the graphs reveals the following:

(1) In general, the effect of transverse roughness is adverse.
(2) The combination of porosity and slip suggests significant reduction in load-carrying capacity.
(3) The increased load-carrying capacity due to magnetization is increased further in the case of negatively skewed roughness. Furthermore, this positive effect is enhanced when negative variance is involved.

Although, a significant adverse effect is induced by some of the parameters, there exists some scope (of course limited) for improving the performance of the bearing system, in the case of negatively skewed roughness, by keeping the porosity and slip at a minimum. In this situation, the role of the step location becomes equally significant.

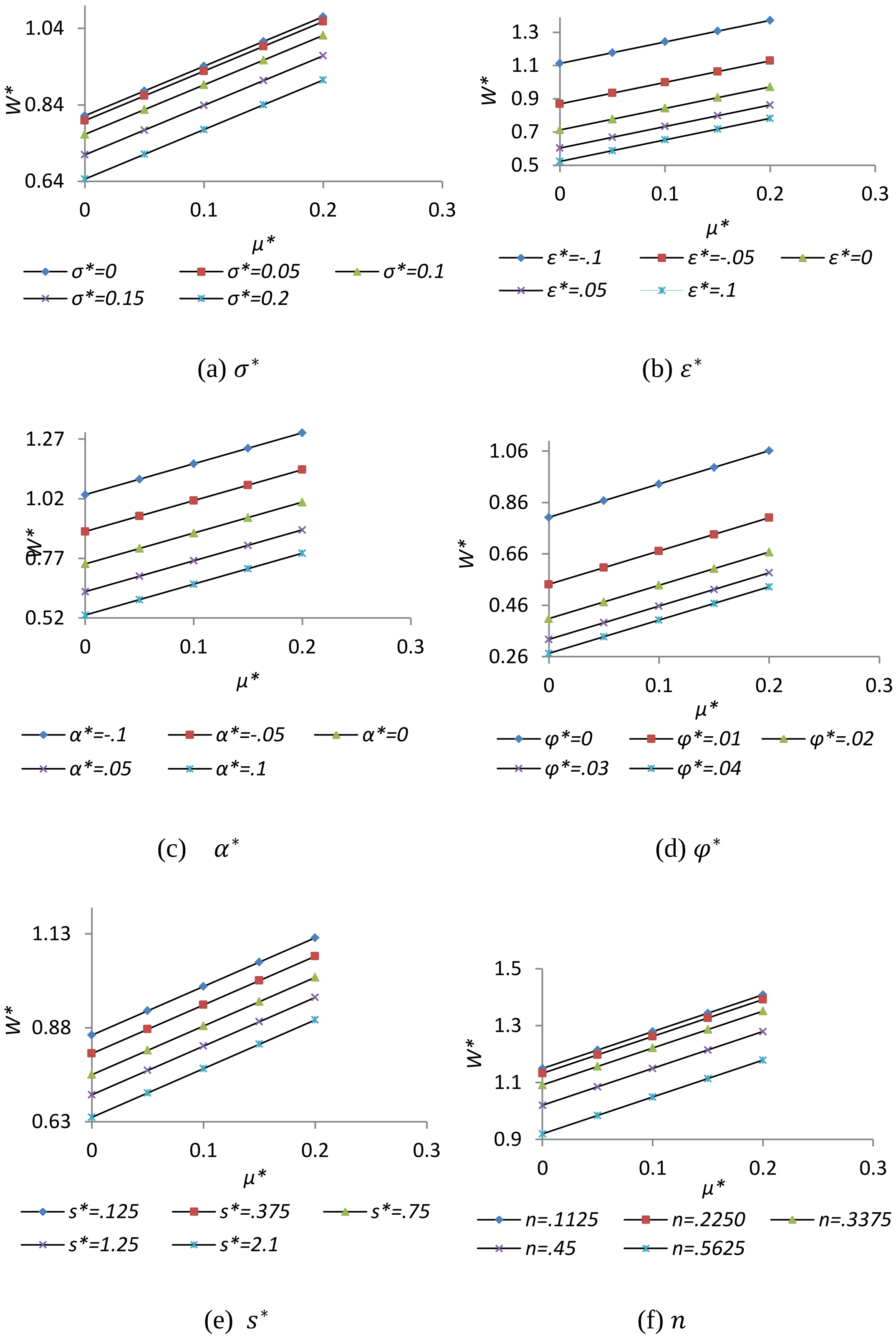
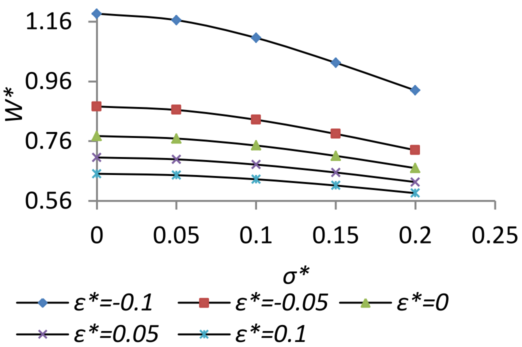
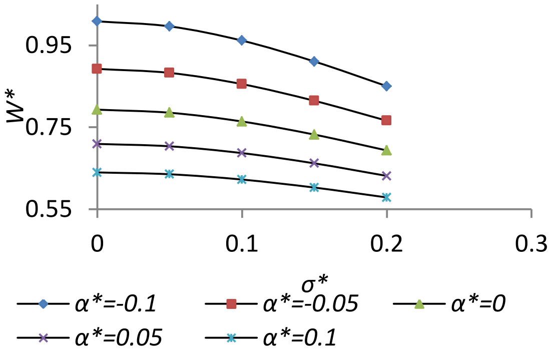


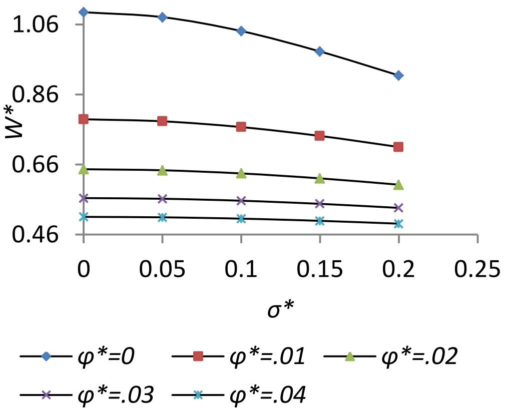
Figure 2. Variation of load-carrying capacity with respect to μ^*.



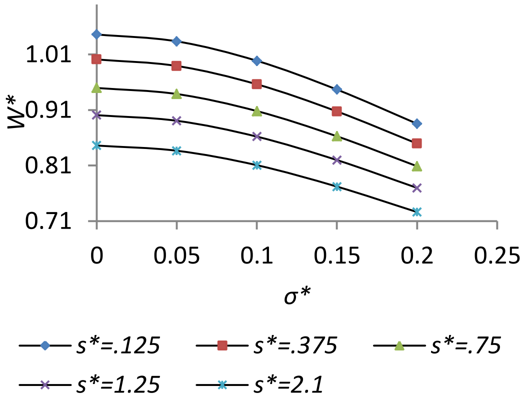
(a) ε^*



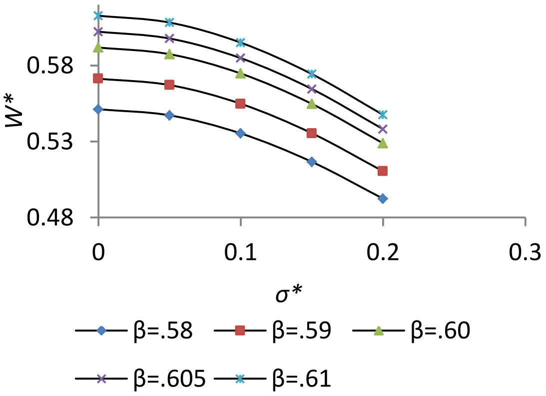
(b) α^*



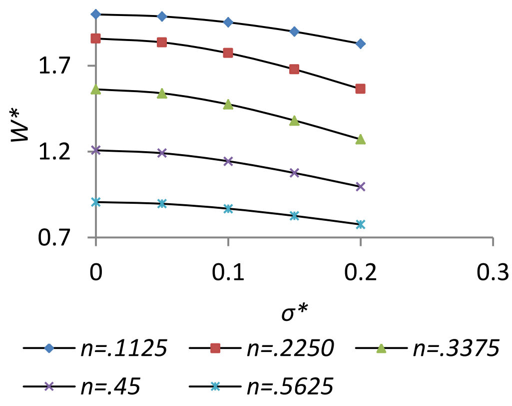
(c) φ^*



(d) s^*

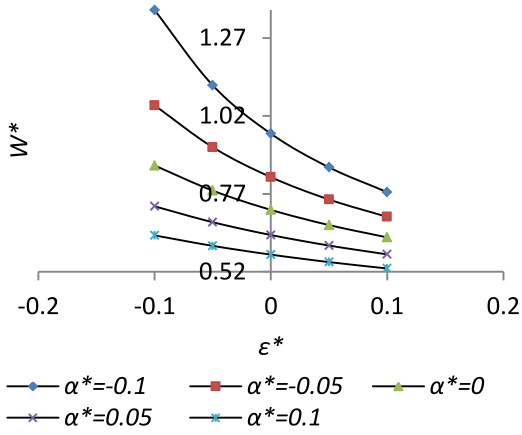


(e) β

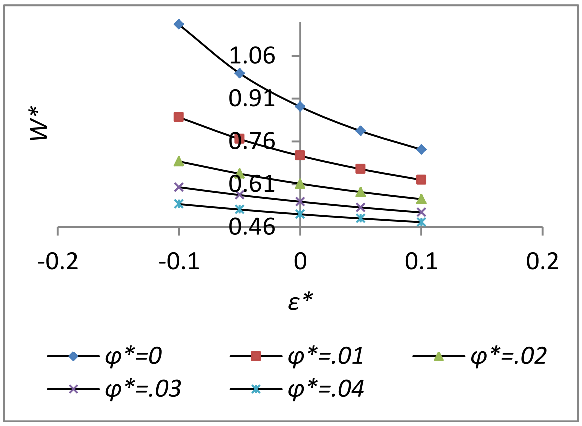


(f) n

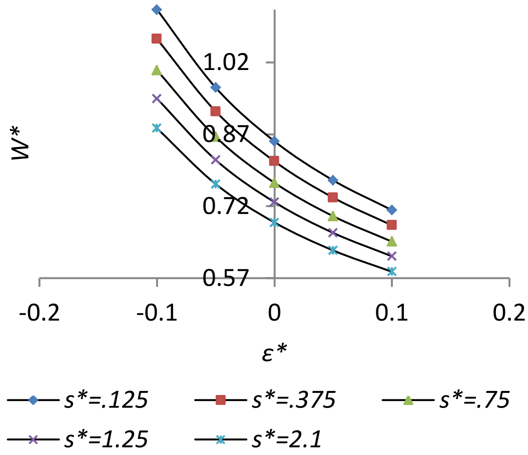
Figure 3. Variation of load-carrying capacity with respect to σ^*.



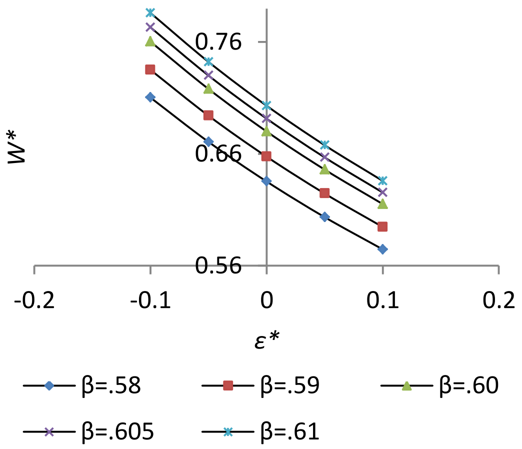
(a) α^*



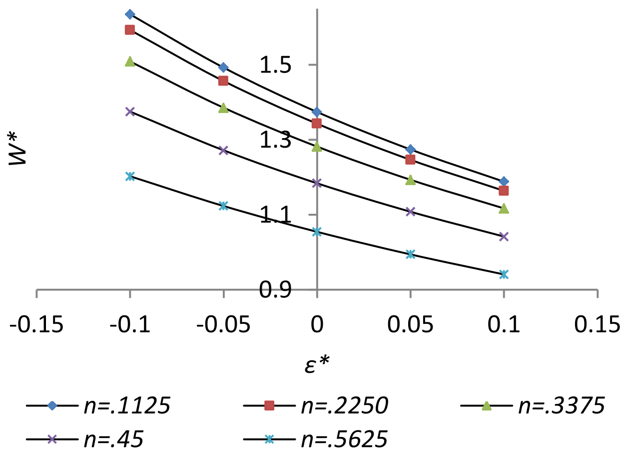
(b) φ^*



(c) s^*

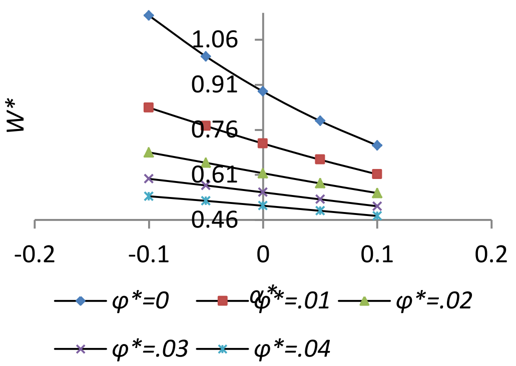


(d) β

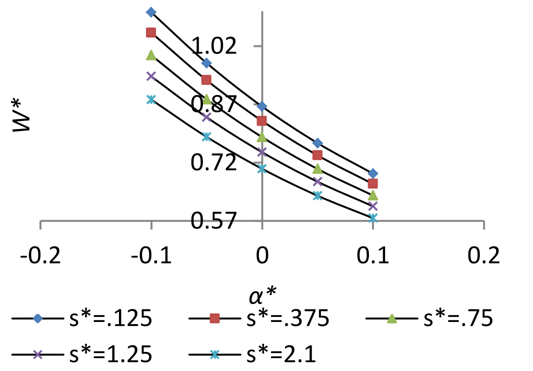


(e) n

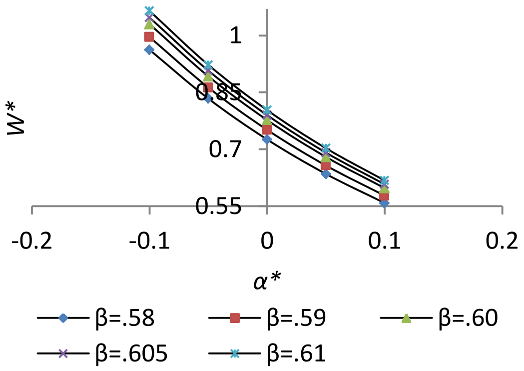
Figure 4. Variation of load-carrying capacity with respect to ε^*.



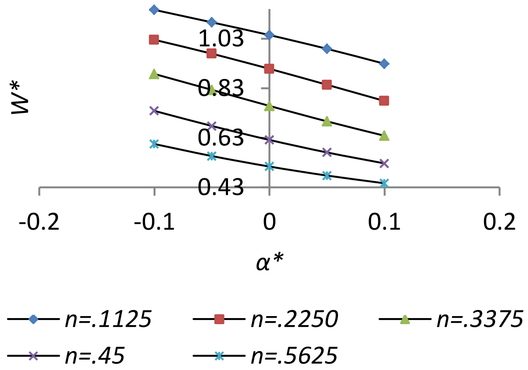
(a) φ^*



(b) s^*

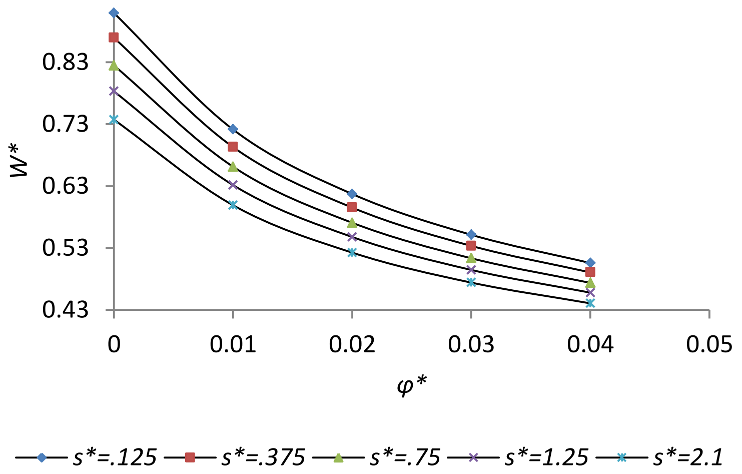


(c) β

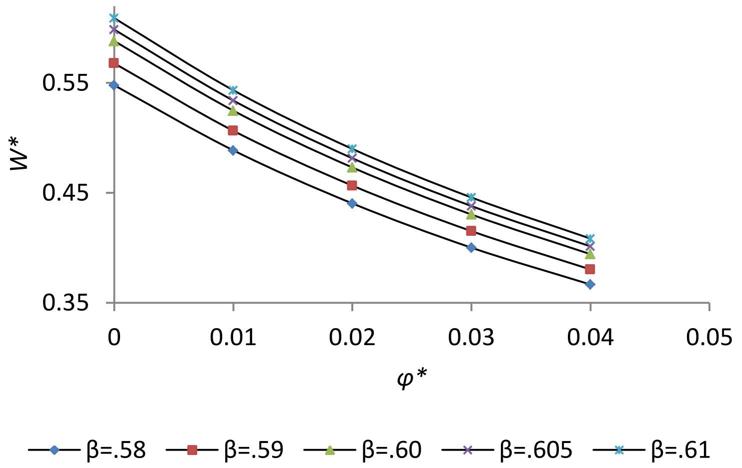


(d) n

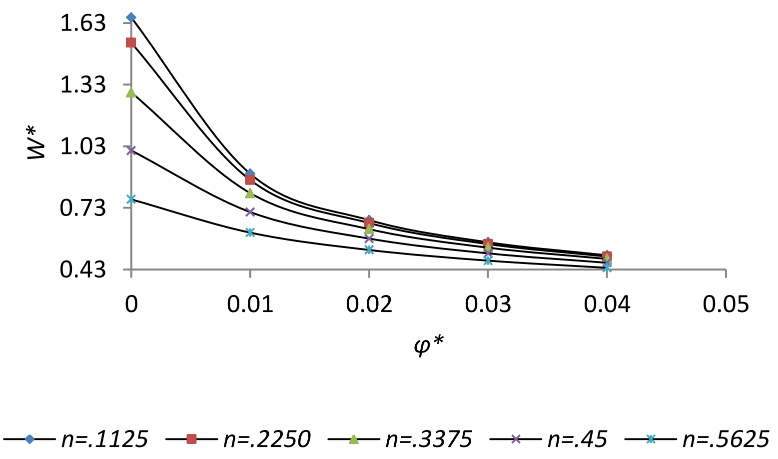
Figure 5. Variation of load-carrying capacity with respect to α^*.



(a) s^*



(b) β



(c) n

Figure 6. Variation of load-carrying capacity with respect to φ^*.

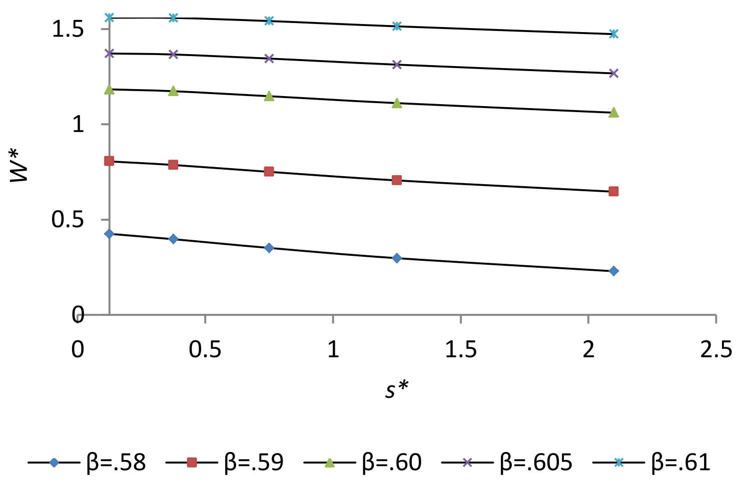


Figure 7. Variation of load-carrying capacity with respect to s^* and β.

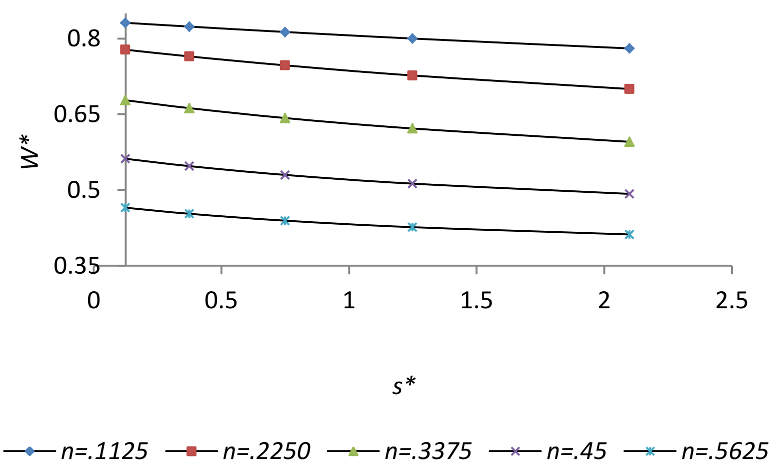


Figure 8. Variation of load-carrying capacity with respect to s^* and n.

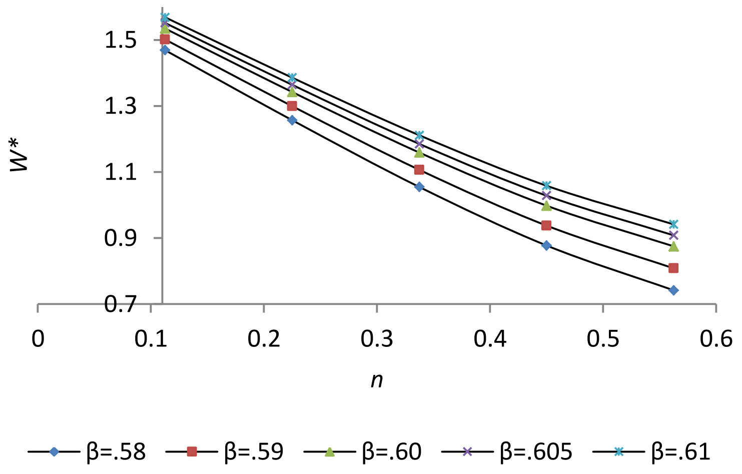


Figure 9. Variation of load-carrying capacity with respect to n and β.

CONCLUSION

A comparison of the present study with previously established results reveals that at best, the slip parameter can decrease the load-carrying capacity by 3.2%. However, the decrease in the load-carrying capacity due to porosity can be up to 7.89%. The reduction in load-carrying capacity due to the standard deviation may be up to 4.7%. It is observed that for an overall improved performance of the bearing the slip should be reduced, no matter how suitable the magnetic strength. This investigation reveals that with regard to the life of the bearing, the roughness aspects must be considered carefully when designing this type of bearing system, even if there is the presence of a suitable magnetic strength and where slip is a minimum. To mitigate the adverse effects of porosity and standard deviation, the step location may offer some help in the case of negatively skewed roughness when variance (-ve) occurs. Furthermore, this type of bearing system supports a load, even in the absence of flow, which is nowhere near true in the case of traditional lubricants. However, for an overall augmented performance of the bearing system, the slip parameter must be minimized.

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NOMENCLATURE

h	fluid film thickness at any point	P^*	dimensionless pressure
n	film thickness ratio		
s	slip parameter	W^*	dimensionless load-carrying capacity
B	bearing length	τ	shear stress
F	frictional force	η	dynamic viscosity of fluid
H	magnitude of magnetic field	φ	porosity
P	lubricant pressure	σ	standard deviation
U	shaft surface speed	ε	skewness
W	load-carrying capacity	α	variance
h_1	inlet film thickness	ρ	fluid density
h_2	outlet film thickness	β	step ratio
h_m	oil film thickness when the pressure is maximum	σ^*	non-dimensional standard deviation
$\bar{q}$	fluid velocity in the film region	ε^*	non-dimensional skewness
s^*	dimensionless slip parameter	α^*	non-dimensional variance
H_m	non-dimensionless film thickness when the pressure is maximum	φ^*	non-dimensional porosity
$\bar{H}$	external magnetic field	μ^*	magnetization parameter
$\bar{M}$	magnetization vector	$\bar{\mu}$	magnetic susceptibility
F^*	dimensionless frictional force	μ_0	permeability of the free space