

OPTIMAL CONTROL STRATEGY FOR LOW SPEED AND HIGH SPEED FOUR-WHEEL-ACTIVE STEERING VEHICLE

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ABSTRACT

In this work, based on the optimal control theory approach, a four-wheel-active steering (4WAS) system is proposed for low speed and high speed applications. A model following the control structure is adopted consisting of a feed-forward and feedback compensation strategy that serves as correction inputs to enhance the vehicle's dynamic behavior. The velocity dependent feed-forward control inputs are based on the driver's steering intention while the feedback control inputs are based on the vehicle's state feedback errors, being the sideslip and yaw rate of the vehicle. Numerical simulations are conducted using the Matlab/Simulink platform to evaluate the control system's performance. The performance of the 4WAS controller is tested in two designated open loop tests, being the constant steer and the lane change maneuver, to evaluate its effectiveness. A comparison with conventional passive front-wheel-steering (FWS) and conventional four-wheel-steering (4WS) systems shows the preminent result performance of the proposed control strategy in terms of the response tracking capability and versatility of the controller to adapt to the system's speed environment. In high speed maneuvers, the improvement in terms of yaw rate tracking error in rms is evaluated and the proposed active steering system considerably beat the other two structures with 0.2% normalized error compared to the desired yaw rate response. Meanwhile, in low speed, turning radius reductions of 25% and 50% with respect to the capability of normal or typical FWS vehicles are successfully achieved.

Keywords: Optimal control; 4WAS; active steer; four-wheel-steering.

INTRODUCTION

The four-wheel-steering (4WS) system is a type of steering system that utilizes all four wheels to turn instead of relying on two wheels to steer as in the conventional steering system. The 4WS system offers improvements in vehicle steering response and high speed stability for swift turning and cornering, as well as a smaller turning radius during low speed maneuverability. It has been in main stream production since the late 1980s, typically in equipping sport cars or off-road trucks[1]. There are two common modes of configuration for 4WS: 1) each front and rear axle wheel turns in the same direction in a high speed environment, and 2) the wheels turn in opposite directions with respect to front and rear axles for low speed applications. The 4WS system can also operate during

special maneuvers, such as lateral parking, zero turning radius and oblique driving[2]. Various types of control strategies have been used to control the 4WS behavior with the aim of improving vehicle maneuverability (i.e. handling) and the comfort (i.e. stability) index of the vehicle. Obviously, an active steering system is a promising solution for such a system issue due to its capability to actively correct the steering input in accordance with the desired response. This can be carried out via a state feedback control approach. However, this cannot be accomplished without a strategic steering direction assignment that could enhance the vehicle's maneuverability and comfort at a given speed range as discussed in [3]. In other words, the direction of the rear steering angle plays a crucial role in order to achieve the desired performance index. The rear steering angle should be made to have the same direction in a high speed range (i.e. positive phase), and vice versa in a low speed region (i.e. opposite direction or negative phase). This could improve vehicle maneuverability as well as enhancing the vehicle's stability performance. For instance, the 4WS system can reduce the vehicle body lean effect during a fast lane change (i.e. in a high speed environment) and has the capability for sharper turns during a low speed maneuver[4].

Recently, a model matching approach with various control theories, such as optimal control[5], sliding mode control[6], and other robust control techniques, have been studied extensively. The performances of such control techniques are evidently superior in the high velocity region. However, the results presented are lacking in terms of controller performance in the low speed region. Therefore, in this paper, the potential of the model matching approach based on optimal control theory for the high speed and low speed modes of four-wheel-active steering (4WAS) is investigated. The term 'active' in 4WAS came from the act of controlling both front and rear wheel angles simultaneously. The versatility of the controller to adapt to variations in the speed range is the main highlight in this work.

VEHICLE MODELING

In this section, a mathematical expression for the nonlinear vehicle model and adopted tire model are presented. The former is based on the well-known Newton's second law of motion, while the latter is utilized for force generation within the rolling mechanism.

Dynamic Vehicle Model

Since the main intention of the work is to focus on the vehicle handling performance, a 7-degree of freedom (DOF) nonlinear vehicle model, adequate for the targeted objective, is considered. A vehicle planar diagram is shown in Figure 1. Such a model consists of four main vehicle dynamic motions: the lateral, longitudinal, yaw and four-wheel rotational speed [7]. The associated mathematical expressions of the considered dynamic are as follows.

Lateral motion:

$$a_y = \frac{\sum F_{yi}}{m} - v_x r \quad (1)$$

Longitudinal motion:

$$a_x = \frac{\sum F_{xi}}{m} + v_y r \tag{2}$$

Yaw motion:

$$I_{zz} \dot{r} = L_f (F_{yFL} + F_{yFR}) - L_r (F_{yRL} + F_{yRR}) + \frac{w}{2} (F_{xFL} - F_{xFR} + F_{xRL} - F_{xRR}) \tag{3}$$

Wheel rotational motion:

$$I_w \dot{\omega}_i = T_i - R_w F_{xi} \tag{4}$$

where v_y , a_y , v_x , a_x , $\dot{\omega}_i$, T_i and r denote the dynamic characteristics of vehicle speed and acceleration in both the lateral and longitudinal directions, the acceleration of each wheel, net torque injected to each wheel and yaw rate response, respectively. Meanwhile m , w , L_f , L_r and R_w denote the vehicle’s physical dimensions, mass, wheel track, distance of front axle to center of gravity, distance of rear to center of gravity, and effective wheel radius, respectively. The terms F_{xi} and F_{yi} in Eqs. (1)–(4), are the tire forces in the X and Y directions, respectively:

$$F_{xi} = F_{ti} \cos \delta_i - F_{si} \sin \delta_i \text{ with } i = FL, FR, RL, RR \tag{5}$$

$$F_{yi} = F_{ti} \sin \delta_i + F_{si} \cos \delta_i \text{ with } i = FL, FR, RL, RR \tag{6}$$

where F_{ti} , F_{si} , and δ_i denote the tire tractive force, tire side force and the steer angle, respectively.

On the other hand, the variation in load transfer within the vehicle is modeled using a quasi-static load model to represent the dynamics of the generated vertical forces at each axle. The mathematical expression of the quasi-static load transfer is written as follows:

$$F_{zFR} = \frac{mgL_r}{2(L_f + L_r)} - \frac{ma_x h}{2(L_f + L_r)} + \frac{ma_y h}{2w} \tag{7}$$

$$F_{zFL} = \frac{mgL_f}{2(L_f + L_r)} - \frac{ma_x h}{2(L_f + L_r)} - \frac{ma_y h}{2w} \tag{8}$$

$$F_{zRR} = \frac{mgL_f}{2(L_f + L_r)} + \frac{ma_x h}{2(L_f + L_r)} + \frac{ma_y h}{2w} \tag{9}$$

$$F_{zRL} = \frac{mgL_r}{2(L_f + L_r)} + \frac{ma_x h}{2(L_f + L_r)} - \frac{ma_y h}{2w} \tag{10}$$

where g and h denote the gravitational acceleration and the height of the vehicle’s center of gravity to ground, respectively.

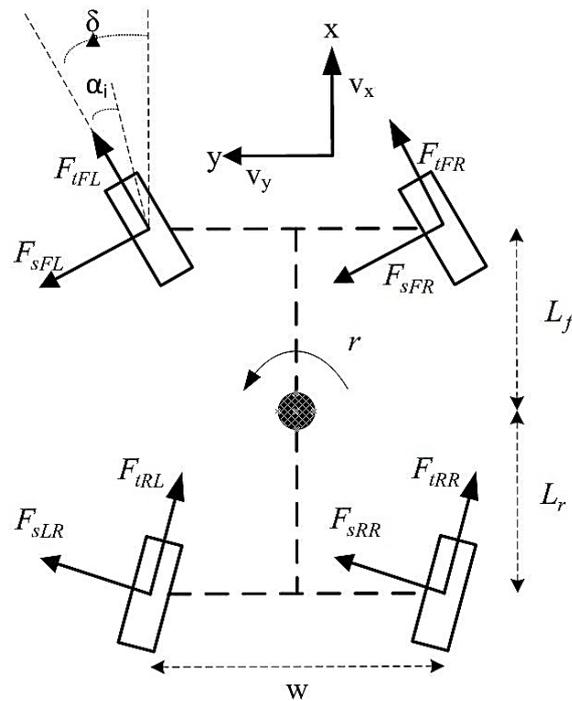


Figure 1. Vehicle planar diagram.

Simplified Calspan Tire Model

Pacejka's semi-empirical model is a well-known tire model which is capable of generating the appropriate force on the tire. This was validated recently in [8] via their simulation and experimental tests conducted for longitudinal vehicle dynamics. However, such a tire model requires quite a number of arbitrary constants which are difficult to determine, but, in earlier work by [9], a simplified Calspan tire model was adopted by the same author. A much simpler model has also matched the performance of Pacejka's model, which is also validated through simulation and experimental work. Hence, this proves that the model is capable of representing the appropriate tire dynamics in the actual environment.

In this work, the tire force response is calculated using the Calspan tire model. The Calspan tire model requires two important inputs: the tire slips in both the longitudinal and lateral motions to generate appropriate resulting forces in the X and Y directions according to the limit of the friction ellipse. In fact, most of the available tire models require those two parameters, which can be calculated based on instantaneous vehicle dynamics. The lateral slip is presented by the tire slip angle α , which is calculated based on the vehicle longitudinal velocity, lateral velocity, yaw rate and the tire steer angle. Meanwhile, the longitudinal tire slip is presented by a ratio as a function of vehicle and wheel velocity. Generally, the slip ratio is calculated in two different modes of operation: acceleration and deceleration. The governing equations of both lateral and longitudinal tire slips are as given by Eqs. (11)–(14):

i) Lateral tire slip (slip angle).

$$\alpha_{FL} = \alpha_{FR} = \delta_F - \arctan\left(\frac{V_y + L_f r}{V_x}\right) \tag{11}$$

and

$$\alpha_{RL} = \alpha_{RR} = \delta_{FR} + \arctan\left(\frac{L_R r - V_y}{V_x}\right) \tag{12}$$

ii) Longitudinal tire slip (slip ratio).

$$l_{accel} = \frac{U_{wi} - R_w w_i}{U_{wi}}, \quad R_w w_i < U_{wi} \quad (\text{acceleration mode}) \tag{13}$$

$$l_{decel} = \frac{R_w w_i - U_{wi}}{U_{wi}}, \quad R_w w_i \geq U_{wi} \quad (\text{deceleration mode}) \tag{14}$$

where U_{wi} is the velocity of the wheel hub in rolling direction with $i=FL, FR, RL$ and RR , which are estimated using the following equations [7]:

$$U_{wFL} = (v_x - 0.5rw) \cos d_f + (v_y + rL_f) \sin d_f \tag{15}$$

$$U_{wFR} = (v_x + 0.5rw) \cos d_f + (v_y + rL_f) \sin d_f \tag{16}$$

$$U_{wRL} = (v_x - 0.5rw) \cos d_r + (v_y - rL_r) \sin d_r \tag{17}$$

$$U_{wRR} = (v_x + 0.5rw) \cos d_r + (v_y - rL_r) \sin d_r \tag{18}$$

These linear velocities are estimated using the integrals of the vehicle longitudinal, lateral and yaw acceleration [7]. With the velocity at each wheel plane of the vehicle are estimated via Eqs. (15)–(18), thus, the longitudinal tire slip at each wheel can be calculated using Eq. (13) and Eq. (14), in which the wheel velocity w_i , is readily available from the wheel dynamics equation given by Eq. (4).

Formerly, the Calspan tire model was first introduced by [10], and has the same purpose as other available tire models. Since the model is a function of composite slips, it leads to a complex and highly nonlinear form of composite force. Consequently, the saturation function was introduced by [11], allow the composite force to be calculated with any variations in normal load and coefficient of friction. Furthermore, this simplified Calspan tire model is also capable of describing the vehicle behavior in any driving scenarios[12]. The governing equations to calculate the tire forces via the Calspan tire model are given by:

$$F_{ti} = F_{x-direction(tractive)} = \frac{F_s k_c s}{\sqrt{k_s \tan^2(a) + k_c s^2}} mF_z \tag{19}$$

and,

$$F_{si} = F_{y-direction(side)} = \frac{F_s k_s \tan(a)}{\sqrt{k_s \tan^2(a) + k_c s^2}} mF_z \tag{20}$$

where m , F_z , F_s , s , and a denote the tire adhesion coefficient, normal force, saturation function, tire longitudinal slip ratio and tire slip angle respectively. Elsewhere, variables k_s and k_c , the longitudinal and lateral stiffness coefficients, respectively, are also being considered in force determination. The stiffness coefficients are equated by considering the tire's physical properties, such as the contact patch, tread, and pressure. Parameter selection and calculation of the saturation function are adopted from the work in [9].

FOUR-WHEEL-ACTIVE STEERING

In this work, the 4WS control structure has been categorized into two conditions: the high speed and low speed control. It consists of two main configurations: the identical wheel turning direction for a high speed maneuver and the opposite wheel turning direction for a low speed maneuver. In order to enable the front and rear wheel angles to turn in similar or different directions, a controller that can switch the direction of the wheel base according to speed is needed. In the high speed 4WAS control, a model following the control structure is adopted to control the yaw rate and sideslip by following the desired model. Meanwhile, in the low speed 4WAS control, a similar control strategy is adopted with a modification to the reference model to enable the desired model to produce a smaller turning radius. Generally, in a vehicle with 4WS capability, the turning radius can be reduced by about 21% to 26%, that is by around a quarter of its original turning radius[13]. This is supported by a report released by the Delphi Automotive System Corp stating that a vehicle equipped with 4WS capability could reduce the turning radius by up to 26% with respect to a normal turning radius[14]. In this paper, a reduction of a quarter and a half in the vehicle's desired turning radius are adopted as the new target references for the low speed controller in accordance with the typical production vehicle. The 50% turning radius is considered as an extreme case scenario where modification of the vehicle's physical chassis design is necessary in order to meet such a requirement. The default desired turning radius is obtained from the steering wheel input commanded by the driver.

Desired Model

In order to improve the handling and stability of the vehicle, two vehicle states, which must be followed for the desired response, are selected: the sideslip and yaw rate response. The desired sideslip response is designed to be zero in steady state at the center of gravity, while the desired yaw rate response is represented by the first order lag. The adopted desired vehicle states are written as follows;

$$X_d = \begin{bmatrix} \beta_d \\ r_d \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{k_r}{1 + \tau_r s} \end{bmatrix} \quad (21)$$

$$k_r = \frac{v}{L(1 + K_{us} v^2)} \delta \quad (22)$$

where k_r , τ_r and K_{us} are the steady state yaw rate response, desired time constant and cornering stability factor, respectively.

On the other hand, for low speed 4WAS control, the main goal is to achieve a smaller turning radius via modifying the desired yaw rate response of the vehicle. As mentioned earlier, this is done by reducing the default turning radius R , by a factor of a quarter and a half. In other words, for low speed 4WAS control, the desired yaw rate with a smaller turning radius will be generated. Eq. (23) expresses the modification that has been made to the targeted turning radius for a reduction of a quarter and a half of the default turning radius, respectively.

$$R_{0.25} = 0.75R = \frac{3}{4}R \text{ and } R_{0.5} = 0.5R = \frac{1}{2}R \tag{23}$$

Generally, for a neutral steer, given that the vehicle wheelbase is physically regarded as a constant, the steering input has an inverse proportional relationship with the turning radius[7]. Based on this relationship, the modified turning radius for the low speed control can be written as;

$$\delta = \frac{L}{R} \text{ where } \delta \propto \frac{1}{R} \tag{24}$$

$$R_{0.25} = \frac{3L}{4\delta} \text{ and } R_{0.5} = \frac{L}{2\delta} \tag{25}$$

Substituting the new target turning radius into Eq. (27) yields the new steady state yaw rate reference and can be written as in Eq. (28).

$$r = \frac{\sqrt{v_x^2 + v_y^2}}{R_{low\ speed}} \text{ where } v = \sqrt{v_x^2 + v_y^2} \tag{26}$$

$$r_{0.25} = \left[\frac{4v}{3L} \right] \delta \text{ and } r_{0.5} = \left[\frac{2v}{L} \right] \delta \tag{27}$$

$$k_{r0.25} = \frac{4v}{3L(1 + K_{us}v^2)} \delta \text{ and } k_{r0.5} = \frac{2v}{L(1 + K_{us}v^2)} \delta \tag{28}$$

The cornering stability factor and the desired time constant are also included in the final form of the first order lag desired yaw rate reference response.

CONTROLLER DESIGN

In this section, the design approach for the control system is described. The controller is designed based on 2-DOF linear vehicle model system consisting of the sideslip and yaw rate response as the state variables. Two input variables, the front and rear wheel steering angles, are also incorporated into the system. The governing equation of the linear vehicle model can be expressed in the following state space form:

$$\dot{x} = Ax + Bu \tag{29}$$

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{-C_f - C_r}{mv} & -1 - \frac{L_f C_f - L_r C_r}{mv^2} \\ -\frac{L_f C_f - L_r C_r}{I_z} & \frac{L_f^2 C_f + L_r^2 C_r}{I_z v} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \frac{C_f}{mv} & \frac{C_r}{mv} \\ \frac{L_f C_f}{I_z} & \frac{L_r C_r}{I_z} \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_r \end{bmatrix} \quad (30)$$

Principally, the control inputs of the system are the summations of the feed-forward and feedback compensation which consist of the nominal and correction terms of both front and rear steering angles. The control structure adopted in this work is as depicted in Figure 2.

$$u = u_{ff} + u_{fb} = \begin{bmatrix} \delta_f \\ \delta_r \end{bmatrix} = \begin{bmatrix} \delta_f \\ k\delta_f \end{bmatrix} + \begin{bmatrix} \Delta\delta_f \\ \Delta\delta_r \end{bmatrix} \quad (31)$$

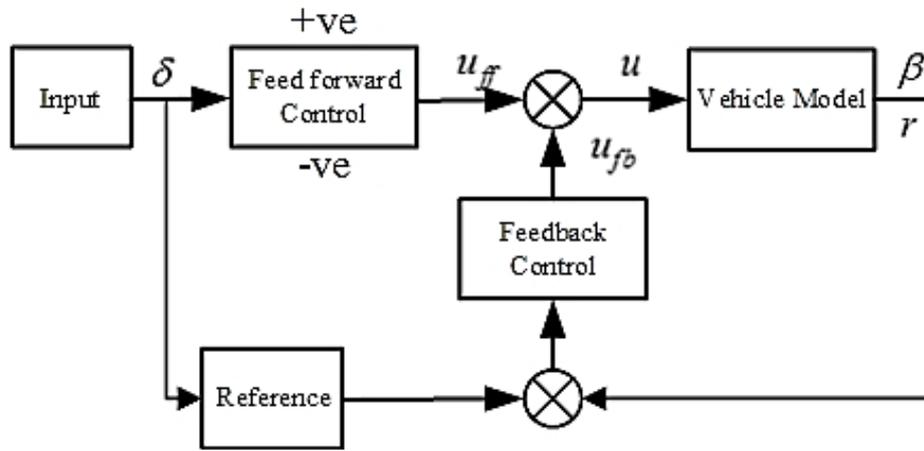


Figure 2. Control structure.

Feed-forward Controller

The main purpose of the feed-forward control is to supply a negative steer (i.e. opposite direction/negative phase) at low speed and positive steering (i.e. same direction/positive phase) at high speed. The rear steering angle is determined based on a velocity function ratio of rear and front steering, as proposed by [3] and further explained in [15], as written in Eq. (32). Hence, the magnitude and direction of the rear steer angle is determined based on the instantaneous velocity ratio function of the vehicle commanded wheel steer input by the front axle.

$$k = \frac{\delta_r}{\delta_f} = \frac{-l_r + \left[\frac{ml_f}{C_r L} \right] v^2}{l_f + \left[\frac{ml_r}{C_f L} \right] v^2} \quad (32)$$

Feedback Controller

The feedback controller is designed utilizing a linear 2-DOF vehicle model system, as mentioned earlier. The purpose of the feedback control law is to compensate the error dynamics which are autonomously or externally generated due to the front steering angle and forces associated with vehicle motion. Based on the linear 2-DOF vehicle model, both state errors are selected in the feedback system in order to track the desired reference response. An optimal control theory is adopted for the feedback control law in order to determine an appropriate gain, thus minimizing the error of the sideslip angle and yaw rate. The error state variable is defined as the difference between the actual vehicle output x , and the reference model output x_d .

$$e = x - x_d = \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} \tag{33}$$

Basically, to construct the feedback compensation, the derivative of the error response of the system is considered and written as;

$$\dot{e} = \dot{x} - \dot{x}_d \tag{34}$$

By substituting the derivative of both actual and desired state equations into Eq. (34), the state space form of error state can be written as follows;

$$\dot{e} = Ae + Bu_{fb} + D_e \tag{35}$$

where D_e is the sum of the third and fourth terms in Eq. (34) which is considered as a steering input dependent lumped disturbance. Hence, the control law for the feedback controller can be written as follows;

$$u_{fb} = -K_{fb}e = - \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} \tag{36}$$

The gain K_{fb} is the feedback matrix which is determined using the linear quadratic regulator (LQR) method by minimizing the cost function which consists of the error states and the feedback control input variables. The two positive definite weighting matrices are selected based on an identity matrix and Bryson's rule, respectively.

NUMERICAL ANALYSIS

In this section, the numerical analysis of the proposed controller strategy is presented and discussed. The main objective of the assessment is to evaluate the handling performance of the vehicle in the event of two designated maneuvers for the high speed and low speed environments, respectively.

Simulation Setup

The proposed control strategies have been numerically simulated using the Matlab/Simulink platform. The analysis consists of two parts: high speed and low speed analysis. The vehicle is assumed to be moving at a constant speed of 10 km/h in the low speed and 80 km/h in the high speed environments, respectively. The vehicle is simulated on a dry asphalt road with a friction coefficient of 0.85. The simulation parameters are as given in Table 1.

For performance evaluation purposes, two other types of vehicle steering system, the passive FWS and the conventional 4WS, are compared with the proposed 4WAS systems. The FWS is assumed to be the uncontrolled vehicle model's behavior based on the input of the front steering angle only. On the other hand, for the conventional 4WS system, the rear steering angle is determined based on the yaw rate state feedback and the front steering angle feed-forward, which is similar to the work in[16]. Open loop lane change (LC) and step steer (SS) maneuvers are selected to evaluate the controller's effectiveness for the high speed and low speed simulations, respectively.

Table 1. Simulation parameters

Parameters for Vehicle Plant, Bicycle Model, and Controller					
Symbols	Description	Value	Symbols	Description	Value
m	Vehicle mass	1289 kg	W	Track width	1.436 m
I_{zz}	Yaw inertia	1627 kg/m ²	R_w	Effective rolling radius	0.35 m
I_w	Wheel inertia	2.1 kg/m ²	C_f	Front cornering stiffness	73520
L_f	Distance front axle to Centre of Gravity	1 m	C_r	Rear cornering stiffness	97058
L_r	Distance rear axle to C.G.	1.454 m	K_{us}	Stability factor	0.005
h	Height from ground to C.G.	0.6 m	τ_r	Time constant	0.0375

High Speed Analysis (LC Maneuver)

In high speed analysis, a 0.5 Hz single period sinusoidal wave with amplitude of ± 0.08 radian (i.e. $\pm 4.58^\circ$) is regarded as the open loop steering input which starts from 1s to 3s[17]. Figures 3 and 4 depict the results of the simulation. Figure 3 shows the comparison of the yaw rate response dynamic of the proposed controller with the other two compared steering systems. Apparently, the 4WAS controller improves the vehicle yaw rate response and is able to track the targeted yaw rate reference. Meanwhile, Figure 4 depicts the vehicle path in the global coordinate position. Based on both results, the proposed 4WAS controller shows promising results, having successfully tracked the desired yaw rate, thus effectively following the desired path given that the vehicle is moving at a constant speed of 80 km/h. It shows that the effectiveness of the proposed controller in a high speed environment could considerably improve the handling performance of the vehicle. This is due to the driver reacting less rapidly

(during the changing motion) in order to achieve the targeted yaw rate with the assistance of the positive phase active rear steering correction[13]. The tracking error for each structure in rms and the normalized rms value with respect to the desired yaw rate response are as in Table 2. Obviously, with a typical FWS without feedback control, the tracking error is expected to be high. In this work, with respect to the adopted parameter and the desired yaw rate response, the normalized tracking error in rms can be reduced to 0.2% compared to the desired response. This shows that the proposed active steering system has promising performance in terms of tracking the desired objective (i.e. yaw rate response).

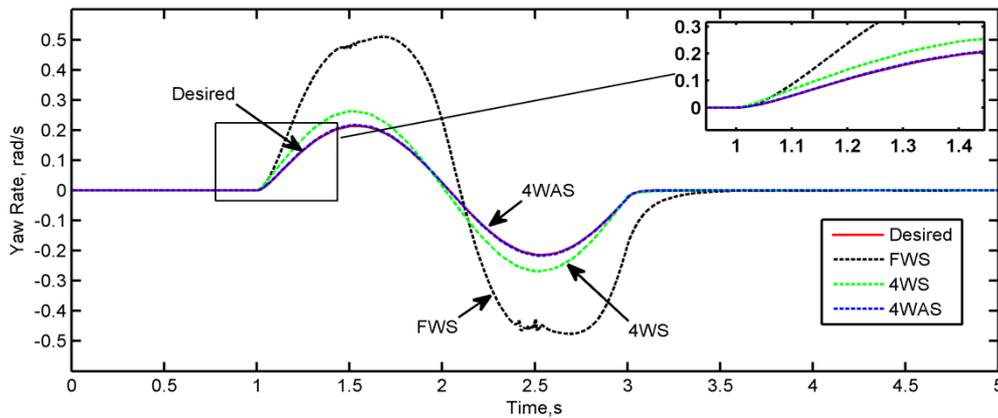


Figure 3. Yaw rate response for LC maneuver.

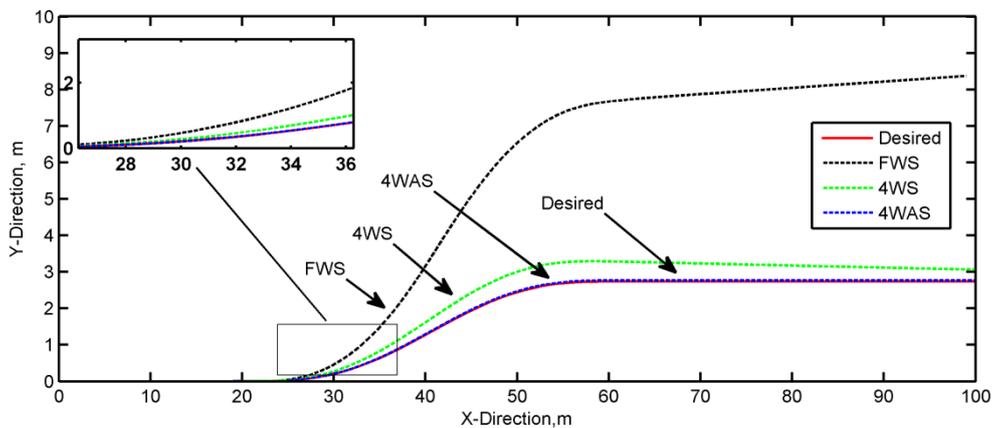


Figure 4. Vehicle path for LC maneuver.

Table 2. Results summary (High speed analysis).

Structure	Yaw rate tracking error, rms	Normalized rms error
FWS	15.05×10^{-3}	0.3536
4WS	23.37×10^{-3}	0.0557
4WAS	0.99×10^{-3}	0.0023

Low Speed Maneuver

On the other hand, in the low speed analysis, the vehicle is subjected to a constant 15° SS input with the vehicle assumed to be moving at a constant speed of 10 km/h. The main objective is to achieve the turning radius requested by the desired control reference with respect to the desired turning radius reduction. In this analysis, the FWS vehicle model response is adopted as the default turning radius (i.e. benchmark) for a typical production vehicle’s turning capability. The results are as depicted and tabulated in Figure 5 and Table 3, respectively.

Figure 5 shows the vehicle turning radius comparison based on the subjected modified yaw rate references in Eq. (28). It can be seen that, in the low speed environment, the proposed 4WAS controller is capable of reducing the vehicle turning radius based on the targeted turning radius as mentioned earlier. Reductions of a quarter and a half with respect to the benchmark turning radius have been achieved successfully. This is due to the assistance of the negative phase rear steer angle (i.e. rear steering in the opposite direction) which can actively correct the steer angle to track the desired yaw rate response. Hence, this will enhance the maneuverability as well as the comfort-and-handling stability, due to the reduced steering wheel rotation in the low speed environment[13]. The results are summarized in Table 3. The table shows that the objective to reduce the turning radius to the desired value is considerably accomplished. However, a slight over-steering effect could be observed in both the targeted turning radius simulations. This is considered insignificant for low speed maneuvers.

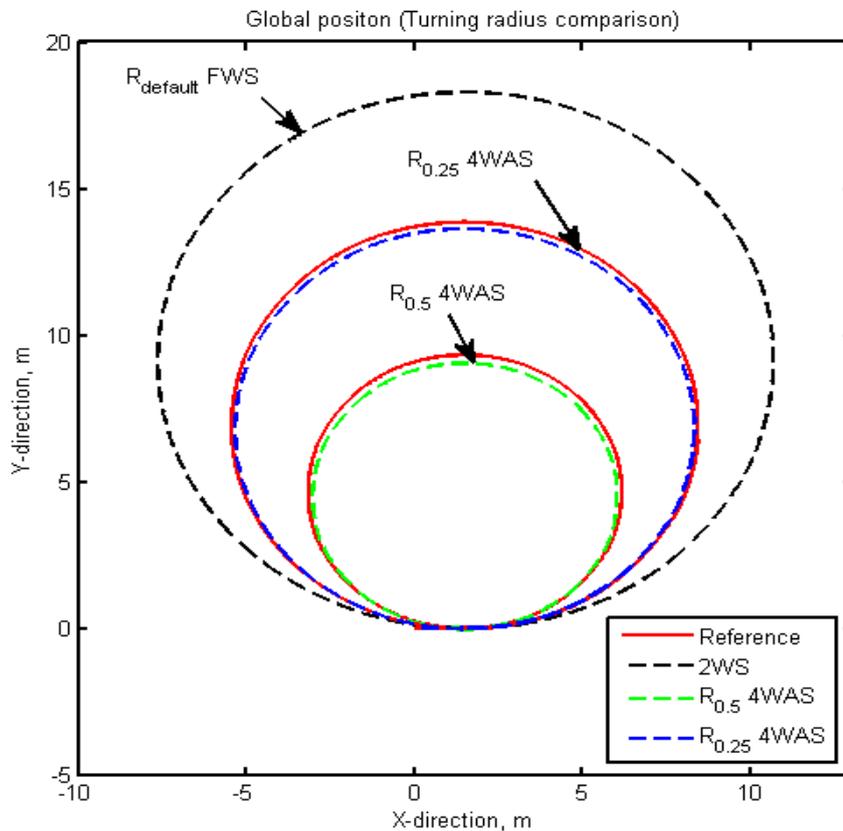


Figure 5. Turning radius comparison.

Table 3. Results summary (Low speed analysis).

Structure	Radius (m)	% Reduction
FWS	9.1 (benchmark)	-
4WAS $R_{0.25}$	6.8	$\approx 25\%$
4WAS $R_{0.5}$	4.53	$\approx 50\%$

CONCLUSIONS

In this paper, a versatile low speed and high speed 4WAS controller, based on optimal control theory, is presented. The proposed controller structure is based on a model control, in which the main objective is to track the desired yaw rate with respect to its desired response in both maneuver conditions. Since the optimal feedback control theory successfully suppressed the vehicle state error, especially the yaw rate response error, the vehicle trajectory in the global position is being promisingly tracked accordingly. Moreover, the gains are optimally determined for both feedback states, therefore, also restraining the vehicle's sideslip error, thus maintaining the vehicle's sideslip generation within its stability region. The capability of rear steering to shift to a different phase mode (i.e. direction) as a function of velocity, also plays a significant role in achieving the desired comfort-and-handling performance. Hence, the main mark of this proposed controller is its capability to adapt to the variation of both low speed and high speed environments. With a given desired reference, the controller is capable of producing a favorable response and coping with the variation in the vehicle's speed environment. Future works shall focus on the variations in the road conditions, in order to investigate the robustness of the proposed control strategy, and the effect of the vehicle's stability in the lateral limit region.

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