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CHEMICAL REACTION EFFECTS ON FLOW PAST AN ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION IN THE PRESENCE OF MAGNETIC FIELD

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ABSTRACT

An exact solution to the problem of unsteady hydromagnetic flow past a uniformly accelerated infinite vertical plate with variable temperature and mass diffusion are presented, taking in to account the homogeneous chemical reaction of first order. The plate temperature as well as the concentration level near the plate increased linearly with time. The dimensionless governing equations are solved using Laplace-transform technique. The velocity, temperature and concentration fields are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, magnetic field parameter, chemical reaction parameter and time. It is observed that the velocity increases with increase of thermal Grashof number and mass Grashof number. It is also observed that the velocity increases with decrease of chemical reaction parameter as well as magnetic field parameter.

Keywords: Accelerated; isothermal; vertical plate; heat and mass transfer; chemical reaction; magnetic field.

INTRODUCTION

Hydromagnetic convection plays an important role in the petroleum industry, geophysics and astrophysics. It also finds applications in many engineering problems such as magnetohydrodynamic (MHD) generators and plasma studies, in the study of geological formations, in the exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites (Pai, 1962). MHD flow has applications in metrology, solar physics and in the movement of the earth's core. It has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. Free convection effects on flow past an accelerated vertical plate with variable suction and a uniform heat flux in the presence of a magnetic field was studied by Raptis, Tzivanidis and Peridikis (1981). Furthermore, MHD effects on flow past an infinite vertical plate for both the classes of impulse as well as accelerated motion of the plate were studied by Raptis and Singh (1983).

Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems the reaction is heterogeneous at an interface and homogeneous in solution (Cussler, 1998; Muthucumaraswamy & Valliammal, 2010; Sathappan & Muthucumaraswamy, 2011). In most chemical reactions the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order when the rate of reaction is directly proportional to the concentration. Chambre and Young (1958) analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das, Deka, and Soundalgekar (1994) studied the effect of a homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Once again, mass transfer effects on a moving isothermal vertical plate in the presence of chemical reaction have been studied by Das, Deka, and Soundalgekar (1999), where the dimensionless governing equations were solved by the usual Laplace-transform technique. Gupta, Pop, and Soundalgekar (1979) studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using the perturbation method. Kafousias and Raptis (1981) extended the above problem to include mass transfer effects subjected to variable suction or injection. Mass transfer effects on flow past a uniformly accelerated vertical plate were studied by Soundalgekar (1982). In addition, the mass transfer effects on flow past an accelerated vertical plate with a uniform heat flux were analyzed by Singh and Singh (1983). The effect of viscous dissipation on Darcy free convection flow over a vertical plate with an exponential temperature was analyzed by Magyari and Rees (2006). The combined effects of heat and mass transfer along a vertical plate in the presence of a transverse magnetic field were studied by Ramesh Babu, and Shankar (2009). Rajput & Kumar (2011) studied the magnetic field effects on flow past an impulsively started vertical plate with variable temperature and mass diffusion. Recently, Vijaya Kumar, Rajasekhara Goud, Varma, and Raghunath (2012) analyzed the radiation effects on MHD flow past a linearly accelerated vertical plate with variable temperature and mass diffusion. Hence here it is proposed to study the first order chemical reaction effects on unsteady flow past a uniformly accelerated infinite vertical plate with variable temperature and mass diffusion in the presence of a transverse magnetic field. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error functions. Such a study would be found useful in chemical process industries such as wire drawing, fiber drawing, polymer production and food processing. The results of the study would provide useful methods for the magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and the magnetic suppression of molten semi-conducting materials.

MATHEMATICAL ANALYSIS

The hydromagnetic flow of a viscous incompressible fluid past a uniformly accelerated isothermal infinite vertical plate with variable temperature and mass diffusion in the presence of a chemical reaction of the first order has been considered. The unsteady flow of a viscous incompressible fluid is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞. The *x*-axis is taken along the plate in the vertically upward direction, and the *y*-axis is taken as being normal to the plate. At time $t' > 0$, the plate is accelerated with a velocity $u = \frac{u_0^3}{\nu} t'$, in its own plane against the force of gravity. The temperature from the plate as well as the concentration levels near the plate are also raised linearly with time, *t*. It is assumed that the effect of viscous dissipation is negligible in the energy equation, and there is a first order chemical reaction between the diffusing species and the fluid. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. Then under the usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{1}$$

$$\rho C_p\frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - K_l C' \tag{3}$$

With the following initial and boundary conditions:

$$\begin{array}{llll} u = 0, & T = T_\infty, & C' = C'_\infty & \text{for all y, } t' \le 0 \\ t' > 0: u = \dfrac{u_0^3}{\nu}t', & T = T_\infty + (T_w - T_\infty)At', & C' = C'_\infty + (C'_w - C'_\infty)At' & \text{at } y = 0 \\ u \to 0, & T \to T_\infty, & C' \to C'_\infty & \text{as } y \to \infty \end{array} \tag{4}$$

where $A = \dfrac{u_0^2}{\nu}$.

By introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0}, t = \frac{u_0^2 t'}{\nu}, Y = \frac{yu_0}{\nu}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, Gr = \frac{g\nu\beta(T_w - T_\infty)}{u_0^3}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gc = \frac{g\nu\beta^*(C'_w - C'_\infty)}{u_0^3} \tag{5}$$

$$M = \frac{\sigma B_0^2\nu}{\rho u_0^2}, K = \frac{K_l\nu}{u_0^2}, Pr = \frac{\mu C_p}{k}, Sc = \frac{\nu}{D}$$

in equations (1) to (4) leads to:

$$\frac{\partial U}{\partial t} = Gr\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} - M\,U \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \tag{8}$$

The initial and boundary conditions in non-dimensional quantities are:

$$\begin{array}{l} U = 0, \quad \theta = 0, \quad C = 0 \quad for\ all\ Y, t \le 0 \\ t > 0: \quad U = t, \quad \theta = t, \quad C = t \quad at \quad Y = 0 \\ U \to 0, \quad \theta \to 0, \quad C \to 0 \quad as \quad Y \to \infty \end{array} \tag{9}$$

METHOD OF SOLUTION

The dimensionless governing Eqs. (6) to (8), subject to the initial and boundary conditions (Eq. (9)), are solved by the usual Laplace-transform technique. The solutions are derived as follows:

$$\theta = t\left[(1+2\eta^2 \Pr)\, erfc(\eta\sqrt{\Pr}) - 2\eta\sqrt{\frac{\Pr}{\pi}}\, \exp(-\eta^2 \Pr)\right] \tag{10}$$

$$\begin{aligned} C = {} & \frac{t}{2}\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc}+\sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc}-\sqrt{Kt})\right] \\ & - \frac{\eta\sqrt{Sct}}{2\sqrt{K}}\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc}+\sqrt{Kt}) - \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc}-\sqrt{Kt})\right] \end{aligned} \tag{11}$$

$$\begin{aligned} U = {} & \left(\frac{t}{2} + t(ad+bc) + c + d\right)\left[\exp(2\eta\sqrt{Mt})\, erfc(\eta+\sqrt{Mt}) + \exp(-2\eta\sqrt{Mt})\, erfc(\eta-\sqrt{Mt})\right] \\ & - \left(\frac{1}{2} + ad + bc\right)\frac{\eta\sqrt{t}}{\sqrt{M}}\left[\exp(-2\eta\sqrt{Mt})\, erfc(\eta-\sqrt{Mt}) - \exp(2\eta\sqrt{Mt})\, erfc(\eta+\sqrt{Mt})\right] \\ & - d\exp(at)\Big[\exp(2\eta\sqrt{(M+a)t})\, erfc(\eta+\sqrt{(M+a)t}) + \\ & \qquad \exp(-2\eta\sqrt{(M+a)t})\, erfc(\eta-\sqrt{(M+a)t})\Big] \\ & - c\exp(bt)\Big[\exp(2\eta\sqrt{(M+b)t})\, erfc(\eta+\sqrt{(M+b)t}) + \\ & \qquad \exp(-2\eta\sqrt{(M+b)t})\, erfc(\eta-\sqrt{(M+b)t})\Big] \\ & - 2d\, erfc(\eta\sqrt{\Pr}) - 2adt\left[(1+2\eta^2 \Pr)\, erfc(\eta\sqrt{\Pr}) - \frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}}\exp(-\eta^2 \Pr)\right] \\ & + d\exp(at)\left[\exp(2\eta\sqrt{\Pr(a)t})\, erfc(\eta\sqrt{\Pr}+\sqrt{at}) + \exp(-2\eta\sqrt{\Pr(a)t})\, erfc(\eta\sqrt{\Pr}-\sqrt{at})\right] \\ & - c(1+bt)\Big[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc}+\sqrt{Kt}) \\ & \qquad + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc}-\sqrt{Kt})\Big] \\ & + \frac{bc\eta\sqrt{Sct}}{2\sqrt{K}}\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc}+\sqrt{Kt}) - \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc}-\sqrt{Kt})\right] \\ & + c\exp(bt)\Big[\exp(2\eta\sqrt{Sc(K+b)t})\, erfc(\eta\sqrt{Sc}+\sqrt{(K+b)t}) + \\ & \qquad \exp(-2\eta\sqrt{Sc(K+b)t})\, erfc(\eta\sqrt{Sc}-\sqrt{(K+b)t})\Big] \end{aligned} \tag{12}$$

where $a = \dfrac{M}{\Pr - 1}, b = \dfrac{M - KSc}{Sc - 1}, c = \dfrac{Gc}{2b^2(1-Sc)}, d = \dfrac{Gr}{2a^2(1-\Pr)}$ and $\eta = \dfrac{Y}{2\sqrt{t}}$.

RESULTS AND DISCUSSION

For the physical interpretation of the problem, numerical computations are carried out for different physical parameters: Gr, Gc, Sc, Pr, M, K and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken as 0.6, which corresponds to water-vapor. The value of the Prandtl number (Pr) is chosen such that it represents water ($Pr = 0.7$). The numerical values of the velocity, concentration and temperature computed for different physical parameters, like thermal Grashof number, mass Grashof number, chemical reaction parameter, magnetic field parameter, Schmidt number, Prandtl number and time, are studied graphically. Figure 1 demonstrates the effect of velocity fields for different values of the chemical reaction parameters $(k = 2.8)$, $Gr = Gc = 5$, $M = 2$ and $t = 0.6$. It is observed that the velocity increases with decreasing values of the chemical reaction parameter. The trend shows that there is a fall in velocity due to the increasing values of the chemical reaction parameter (Das, Deka, & Soundalgekar, 1994).

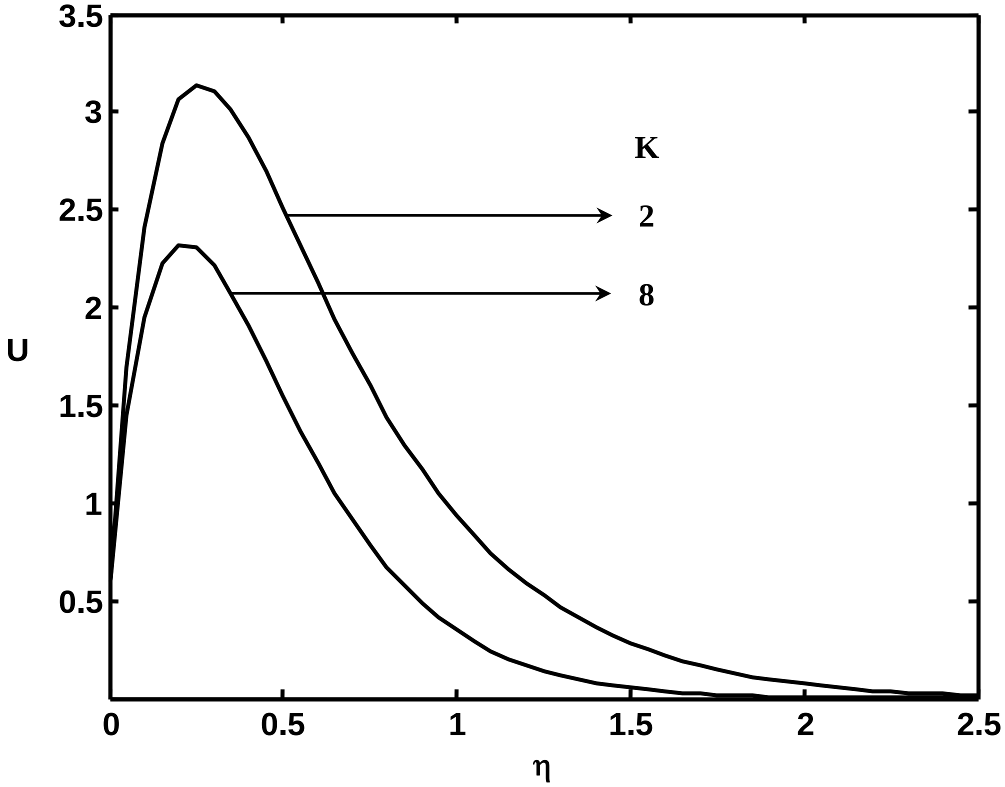

Figure 1. Velocity profile for different K

Figure 2 illustrates the effects of the magnetic field parameter on the velocity when $(M = 1.5, 1.8, 2)$, $Gr = Gc = 5$, $K = 2$ and $t = 0.3$. It was observed that the velocity increases with decreasing values of the magnetic field parameter. This shows that the increase in the magnetic field parameter leads to a fall in the velocity. The result agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow (Raptis & Singh, 1983). The velocity profiles for different values of the thermal Grashof number $(Gr = 2, 5, 10)$, mass Grashof number $(Gc = 5, 10, 15)$, $K = 2$, $M = 2$ and $t = 0.2$ are presented in Figure 3. It was observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number (Das et al., 1994).

The velocity profiles for different values of time $(t = 0.1, 0.2, 0.4)$, $K = 2$, $Gr = Gc = 2$ and $M = 2$ are presented in Figure 4. It is observed that the velocity increases with increasing values of time t (Das et al., 1999). The effect of concentration profiles for different values of the chemical reaction parameter and time are presented in Figure 5. The effect of the chemical reaction parameter is dominant in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the chemical reaction parameter. It is observed that the concentration increases with decreasing chemical reaction (Das et al., 1994). The temperature profiles for air ($Pr = 0.71$) and water ($Pr = 7.0$) are examined, and are shown in Figure 6. It is observed that heat transfer is greater in air than in water. It is clear there is a more rapid drop in temperature in water compared to that in air (Das et al., 1994).

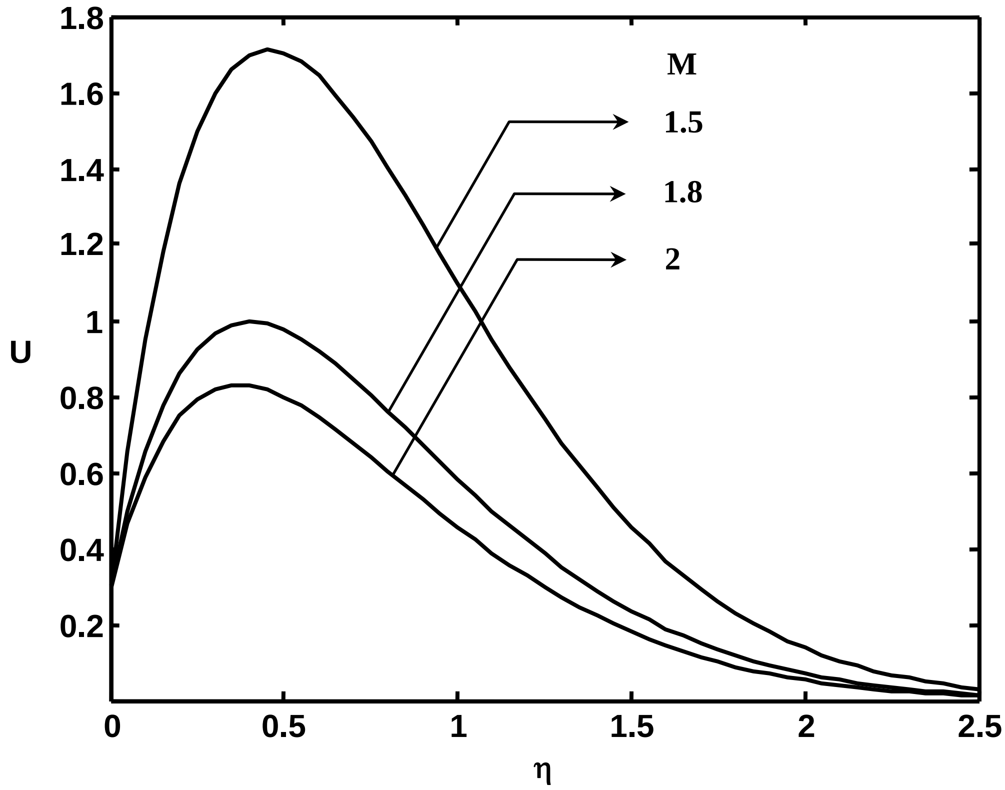

Figure 2. Velocity profile for different M.

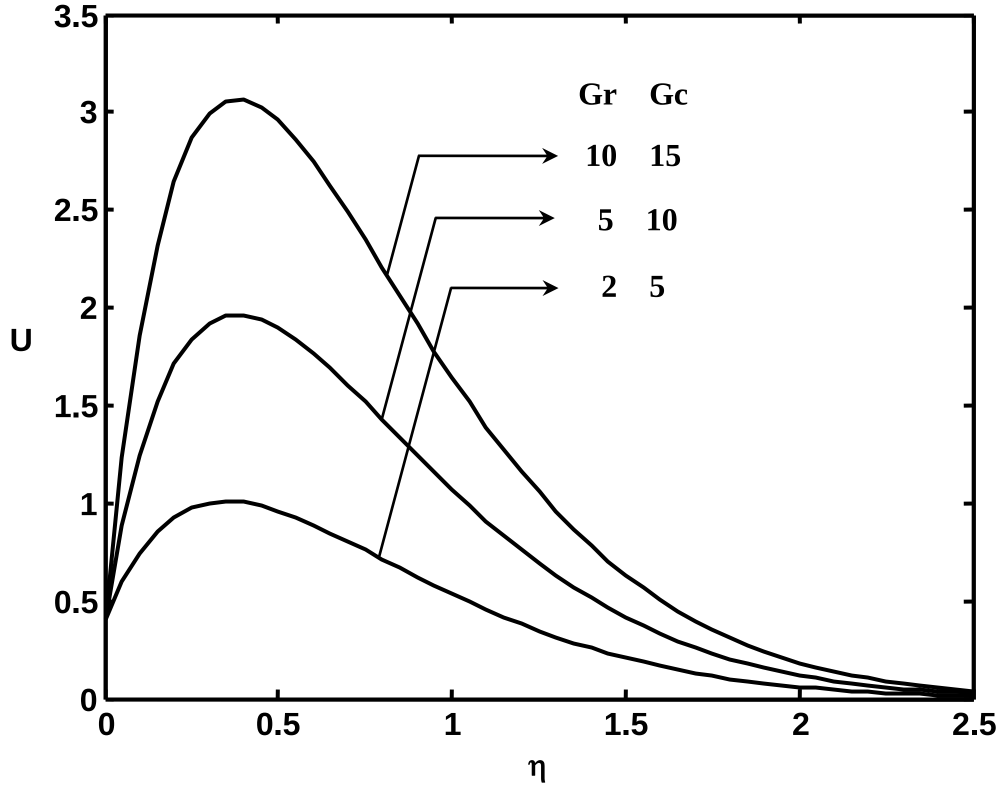

Figure 3. Velocity profile for different Gr, Gc

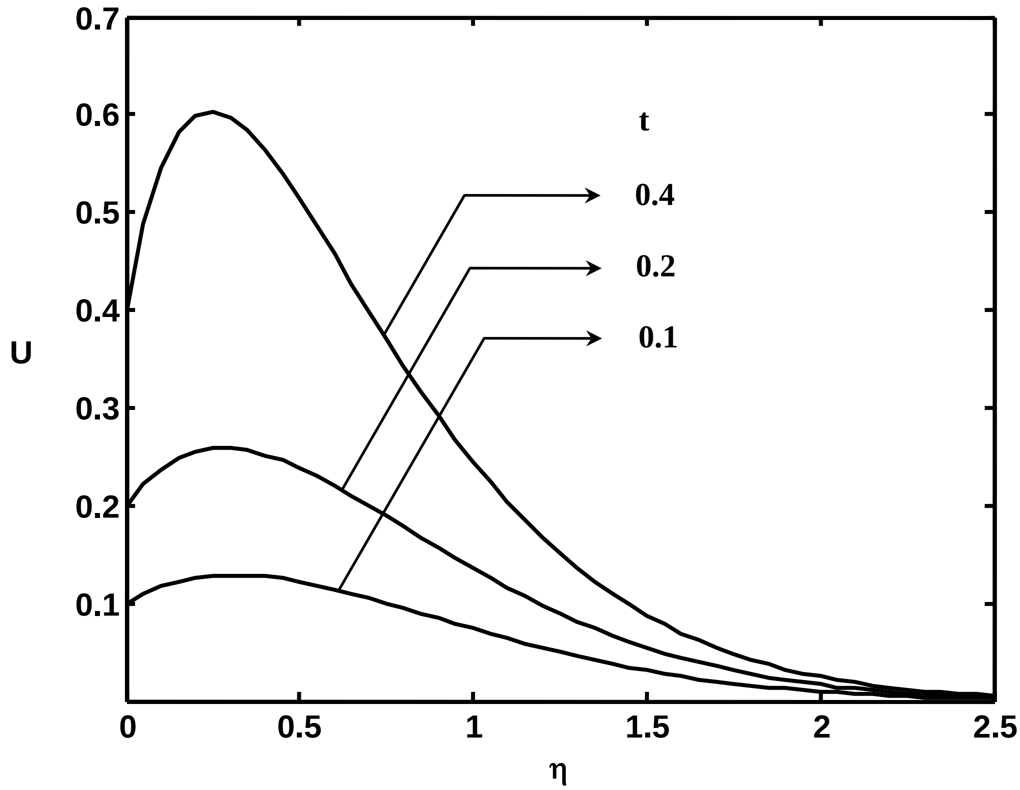

Figure 4. Velocity profile for different t

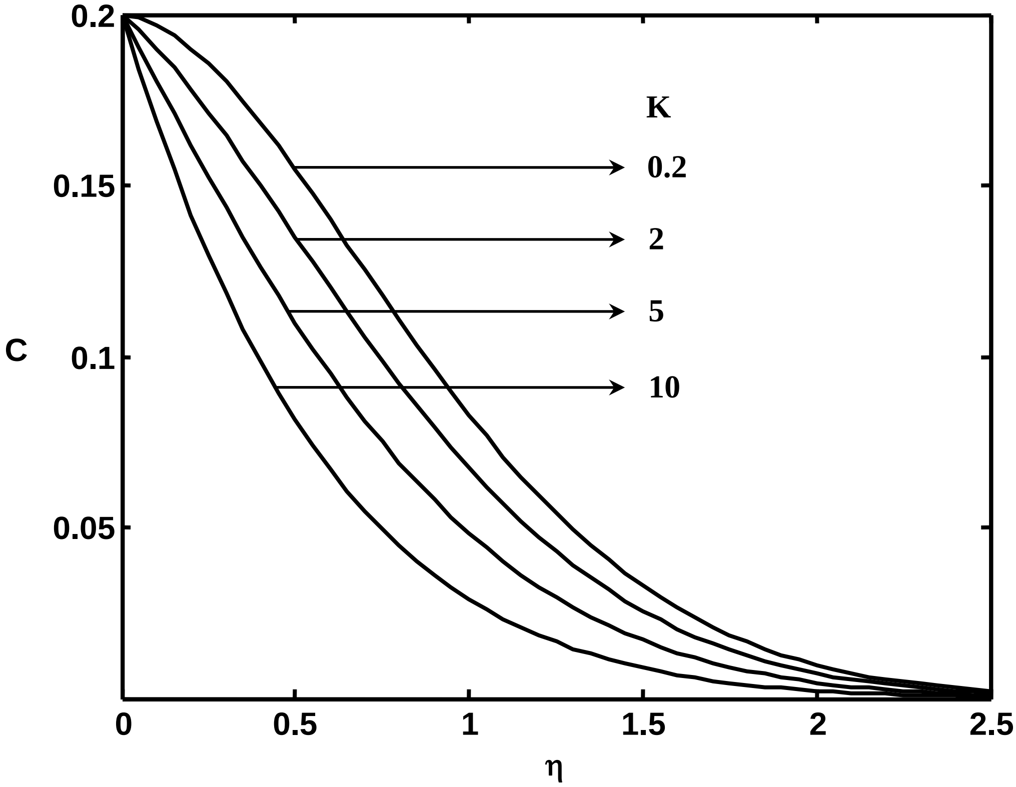

Figure 5. Concentration profile for different K

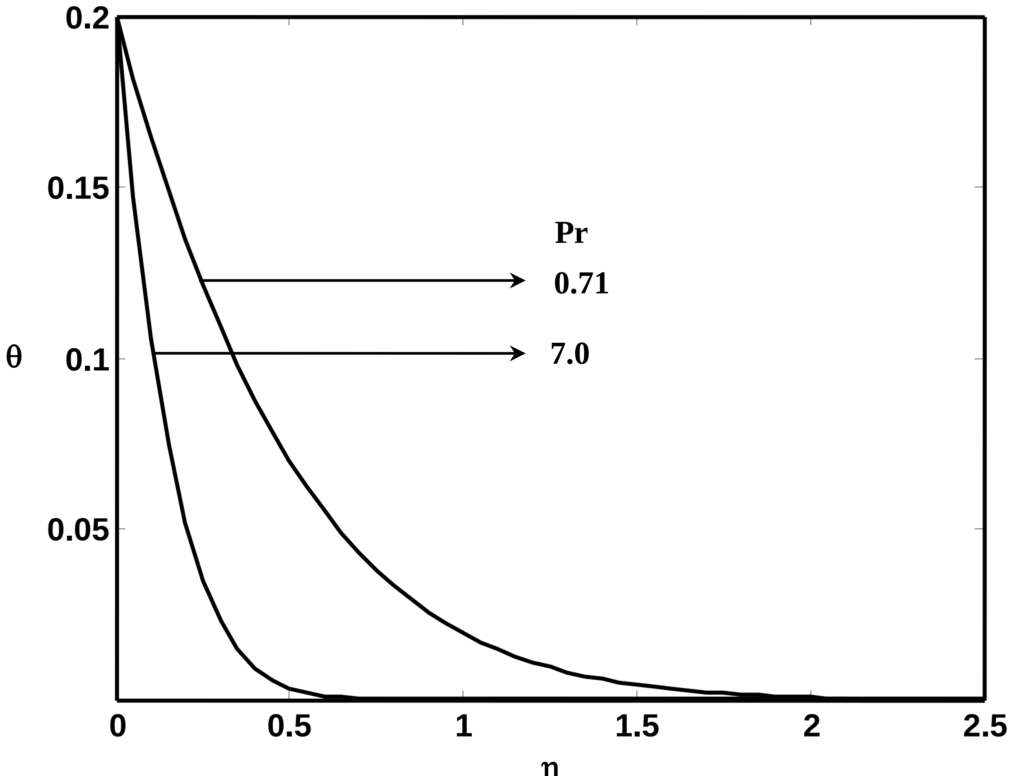

Figure 6. Temperature profile for different Pr

CONCLUSION

The precise analysis of hydromagnetic flow past a uniformly accelerated infinite vertical plate with variable temperature and mass diffusion, in the presence of homogeneous chemical reaction of first order, has been undertaken. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different physical parameters like thermal Grashof number, mass Grashof number, chemical reaction parameter, magnetic field parameter and time are studied graphically. It is observed that the velocity increases with increasing values of Gr, Gc and t. But the trend is reversed with respect to the chemical reaction parameter and magnetic field parameter.

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