Journal of Mechanical Engineering and Sciences (JMES)
ISSN (Print): 2289-4659; e-ISSN: 2231-8380; Volume 3, pp. 346-355, December 2012

DOI: http://dx.doi.org/10.15282/jmes.3.2012.11.0033

MASS TRANSFER EFFECTS ON ACCELERATED VERTICAL PLATE IN A ROTATING FLUID WITH FIRST ORDER CHEMICAL REACTION

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ABSTRACT

The precise analysis of the rotation effects on the unsteady flow of an incompressible fluid past a uniformly accelerated infinite vertical plate with variable temperature and mass diffusion has been undertaken, in the presence of a homogeneous first order chemical reaction. The dimensionless governing equations are solved using the Laplace-transform technique. The plate temperature as well as the concentration near the plate increase linearly with time. The velocity profiles, temperature and concentration are studied for different physical parameters, like the chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number and time. It is observed that the velocity increases with increasing values of thermal Grashof number or mass Grashof number. It is also observed that the velocity increases with decreasing rotation parameter Ω.

Keywords: Rotation; accelerated; isothermal; vertical plate; heat transfer; mass diffusion; chemical reaction.

INTRODUCTION

In many chemical engineering processes there is a chemical reaction between the foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications; viz. polymer production, manufacturing of ceramics or glassware and food processing. Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction (Sathappan & Muthucumaraswamy, 2011). In well-mixed systems the reaction is heterogeneous, if it takes place at an interface and is homogeneous, if it takes place in solution. In most chemical reactions the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order if the rate of reaction is directly proportional to the concentration.

Chambre and Young (1958) analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das, Deka, and Soundalgekar (1994) studied the effect of a homogeneous first order chemical reaction on flow past an impulsively started infinite vertical plate with a uniform heat flux and mass transfer. Furthermore, Das, Deka, and Soundalgekar (1999) studied the effects of mass transfer on a moving isothermal vertical plate in the presence of a chemical reaction. The dimensionless governing equations were solved by the usual Laplace-transform technique. Gupta, Pop, and Soundalgekar (1979) studied the free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using the perturbation method.

Kafousias and Raptis (1981) extended the above problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of a magnetic field were studied by Raptis, Tzivanidis, and Peridikis (1981).

MHD effects on flow past an infinite vertical plate for both classes of impulse as well as the accelerated motion of the plate was studied by Raptis and Singh (1981). The mass transfer effects on flow past a uniformly accelerated vertical plate were studied by Soundalgekar (1982). Jha and Prasad (1990) analyzed the mass transfer effects on flow past an accelerated infinite vertical plate with heat sources. Singh (1984) studied MHD flow past an impulsively started vertical plate in a rotating fluid. Rotation effects on hydromagnetic free convective flow past an accelerated isothermal vertical plate were studied by Raptis and Singh (1985). Recently, the rotation effects on flow past a vertical plate in the presence of thermal radiation was analyzed by Vijayalakshmi (2009). Hence, a study on the chemical reaction effects on flow past an accelerated vertical plate with variable temperature and mass diffusion in the presence of a rotating fluid is proposed. In which the dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error functions. Such a study will be found useful in the control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and meteorology.

GOVERNING EQUATIONS

Consider the unsteady flow of an incompressible fluid past the uniformly accelerated motion of an isothermal vertical infinite plate when the fluid and the plate rotate as a rigid body with a uniform angular velocity Ω' about the z′-axis. Initially the temperature of the plate and concentration near the plate are assumed to be T_∞ and C'_∞. At time $t' > 0$, the plate starts moving with a velocity $u = u_0 t'$ in its own plane against the gravitational field. The plate temperature and the concentration near the plate are raised linearly with time t. Since the plate occupying plane $z' = 0$ is of infinite extent, all physical quantities depend only on z′ and t′. Then the unsteady flow is governed by the usual Boussinesq approximation in dimensionless form as follows:

$$\frac{\partial U}{\partial t} - 2\Omega V = Gr\theta + GcC + \frac{\partial^2 U}{\partial Z^2} \tag{1}$$

$$\frac{\partial V}{\partial t} + 2\Omega U = \frac{\partial^2 V}{\partial Z^2} \tag{2}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Z^2} \tag{3}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Z^2} - KC \tag{4}$$

With the following initial and boundary conditions:

$$\begin{array}{llllll} & u = 0, & T = T_\infty, & C' = C'_\infty & \textit{for all} & y, t' \le 0 \\ t' > 0: & u = u_0 t', & T = T'_\infty + (T'_w - T'_\infty)At', & C' = C'_\infty + (C'_w - C'_\infty)At' & \textit{at} & y = 0 \\ & u \to 0 & T \to T_\infty, & C' \to C'_\infty & \textit{as} & y \to \infty \end{array} \tag{5}$$

On introducing the following non-dimensional quantities:

$$U = \frac{u}{(\nu u_0)^{1/3}}, \quad V = \frac{v}{(\nu u_0)^{1/3}}, \quad t = t'\left(\frac{u_0{}^2}{\nu}\right)^{1/3}, \; Z = z\left(\frac{u_0}{\nu^2}\right)^{1/3}, \tag{6}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \; Gr = \frac{g\beta(T_w - T_\infty)}{u_0}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \; Gc = \frac{g\beta^*(C'_w - C'_\infty)}{u_0}$$

$$\Pr = \frac{\mu C_p}{k}, \; Sc = \frac{\nu}{D}, \; K = K_l\left(\frac{\nu}{u_0^2}\right)^{\frac{1}{3}}$$

where, $A = \left(\frac{u_0{}^2}{\nu}\right)^{1/3}$.

The rotating free-convection flow past an accelerated vertical plate is described by coupled partial differential equations (1) to (4), with the prescribed boundary conditions of Eq. (5). To solve Eqs. (1) and (2) we introduce a complex velocity q = U + iV, Eqs. (1) and (2) can be combined into Eq. (7):

$$\frac{\partial q}{\partial t} = Gr\theta + GcC + \frac{\partial^2 q}{\partial Z^2} - mq \tag{7}$$

The initial and boundary conditions in non-dimensional quantities are:

$$\begin{array}{lllll} & q = 0, & \theta = 0, & C = 0 & \text{for all } Z, t \leq 0 \\ t>0; & q = t, & \theta = t, & C = t & \text{at } Z = 0 \\ & q \to 0, & \theta \to 0, & C \to 0 & \text{as } Z \to \infty \end{array} \tag{8}$$

where, $m = 2i\Omega$.

METHOD OF SOLUTION

The dimensionless governing equations (3), (4) and (7), subject to the initial and boundary conditions (8), are solved by the usual Laplace-transform technique, and the solutions are derived as follows:

$$q = \left(\frac{t}{2} + c + d + (ac+bd)t\right)\left[\exp\,(2\eta\sqrt{2i\Omega t})\, erfc\,(\eta + \sqrt{2i\Omega t}) + \exp(-2\eta\sqrt{2i\Omega t})\, erfc\,(\eta - \sqrt{2i\Omega t})\right]$$
$$-\eta\sqrt{\frac{t}{m}}\left[\frac{1}{2} + ac + bd\right]\left[\exp\,(-2\eta\sqrt{2i\Omega t})\, erfc\,(\eta - \sqrt{2i\Omega t}) - \exp\,(2\eta\sqrt{2i\Omega t})\, erfc\,(\eta + \sqrt{2i\Omega t})\right]$$
$$-2c\; erfc\left(\eta\sqrt{\Pr}\right) - c\exp\,(at)[\exp\,(2\eta\sqrt{(2i\Omega + a)\,t})\, erfc\,(\eta + \sqrt{(2i\Omega + a)\,t})$$
$$+\exp\,(-2\eta\sqrt{(2i\Omega + a)\,t})\, erfc\,(\eta - \sqrt{(2i\Omega + a)\,t})]$$
$$-d\exp\,(bt)[\exp\,(2\eta\sqrt{(2i\Omega + b)\,t})\, erfc\,(\eta + \sqrt{(2i\Omega + b)\,t})$$
$$+\exp\,(-2\eta\sqrt{(2i\Omega + b)\,t})\, erfc\,(\eta - \sqrt{(2i\Omega + b)\,t})]$$
$$+c\exp(at)\,[\exp\left(2\eta\sqrt{at\Pr}\right) erfc\left(\eta\sqrt{\Pr} + \sqrt{at}\right)$$
$$+\exp\left(-2\eta\sqrt{at\Pr}\right) erfc\left(\eta\sqrt{\Pr} - \sqrt{at}\right)]$$
$$+d\exp(bt)\,[\exp\left(-2\eta\sqrt{Sc(K+b)t}\right) erfc\left(\eta\sqrt{Sc} - \sqrt{(K+b)t}\right)$$
$$+\exp\left(2\eta\sqrt{Sc(K+b)t}\right) erfc\left(\eta\sqrt{Sc} + \sqrt{(K+b)t}\right)]$$
$$-d\,(1+bt)[\,\exp\,(2\eta\sqrt{ScKt})\, erfc\,(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp\,(-2\eta\sqrt{ScKt})\, erfc\,(\eta\sqrt{Sc} - \sqrt{Kt})]$$
$$-2act\,[(1+2\eta^2\Pr) erfc(\eta\sqrt{\Pr}) - \frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}}\exp(-\eta^2\Pr)]$$
$$+bd\,\frac{\eta\sqrt{Sct}}{\sqrt{K}}[\exp\,(-2\eta\sqrt{ScKt})\, erfc\,(\eta\sqrt{Sc} - \sqrt{Kt}) - \exp\,(2\eta\sqrt{ScKt})\, erfc\,(\eta\sqrt{Sc} + \sqrt{Kt}) \qquad (9$$

$$\theta = t\left\{\left(1+2\eta^2\Pr\right) erfc(\eta\sqrt{\Pr}) - \frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}}\exp(-\eta^2\Pr)\right\} \qquad (10)$$

$$C = \frac{t}{2}\,[\exp(2\eta\sqrt{ScKt})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{ScKt})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})$$
$$-\frac{\eta\sqrt{Sct}}{2\sqrt{K}}\,[\exp(-2\eta\sqrt{ScKt})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt}) - \exp(2\eta\sqrt{ScKt})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt})\,] \qquad (11)$$

where $a = \dfrac{m}{\Pr - 1}$, $b = \dfrac{m - KSc}{Sc - 1}$, $c = \dfrac{G_r}{2a^2(1-\Pr)}$, $d = \dfrac{G_C}{2b^2(1-Sc)}$ and $\eta = \dfrac{Z}{2\sqrt{t}}$

In order to obtain a physical insight into the problem, the numerical values of q have been computed from Eq. (9). While evaluating this expression it is observed that the argument of the error function is complex and, hence, we have separated it into real and imaginary parts using the following formula:

$$erf(a+ib) = erf(a) + \frac{\exp(-a^2)}{2a\pi}\left[1-\cos(2ab)+i\sin(2ab)\right]$$
$$+\frac{2\exp(-a^2)}{\pi}\sum_{n=1}^{\infty}\frac{\exp(-n^2/4)}{n^2+4a^2}\left[f_n(a,b)+ig_n(a,b)\right]+\in(a,b)$$

where, $f_n = 2a - 2a\cosh(nb)\cos(2ab) + n\sinh(nb)\sin(2ab)$

$$g_n = 2a\cosh(nb)\sin(2ab) + n\sinh(nb)\cos(2ab)$$
$$\left|\in(a,b)\right| \approx 10^{-16}\left|erf(a+ib)\right|$$

RESULTS AND DISCUSSION

For a physical understanding of the problem, numerical computations are carried out for different physical parameters, Gr, Gc, Sc, Pr, m and t, on the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6, which corresponds to water-vapor. Also, the values of the Prandtl number Pr are chosen such that they represent air (Pr = 0.71) and water (Pr = 7.0). The numerical values of the velocity, temperature and concentration fields are computed for different physical parameters, like Prandtl number, rotation parameter, thermal Grashof number, mass Grashof number, Schmidt number and time. The temperature profiles are calculated for water and air from equation (10), and these are presented in Figure 1**.** The effect of the Prandtl number plays an important role in the temperature field. It is observed that the temperature increases with decreasing Prandtl number, for air (Pr = 0.71) and water (Pr = 7.0). This shows that the heat transfer is greater in air than in water.

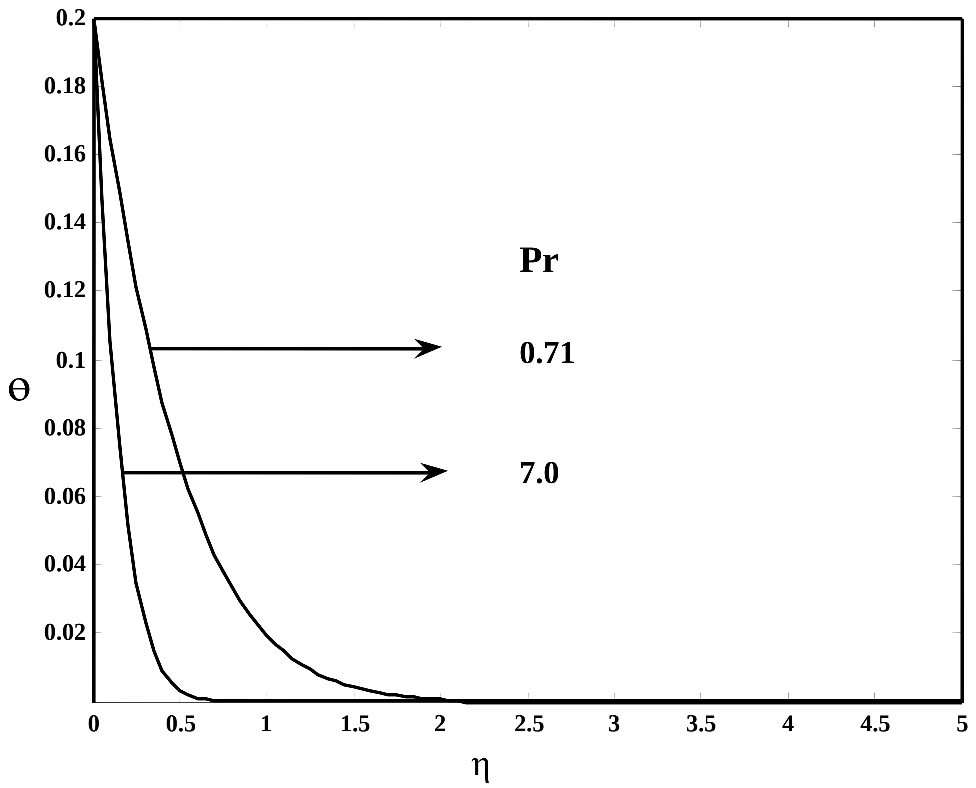

Figure 1. Temperature profiles for different Pr.

Figure 2 represents the effect of the concentration profiles for different Schmidt numbers (Sc = 0.16, 0.3, 0.6, 2.01), K=0.2 and t=0.2. The effect of concentration is significant in the concentration field. The profiles present the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of Schmidt number. The effect of the concentration profiles for different values of the chemical reaction parameter (K = 0.2, 0.6, 1) and t = 0.2 are shown in Figure 3. It is observed that the wall concentration increases with decreasing values of the chemical reaction parameter K.

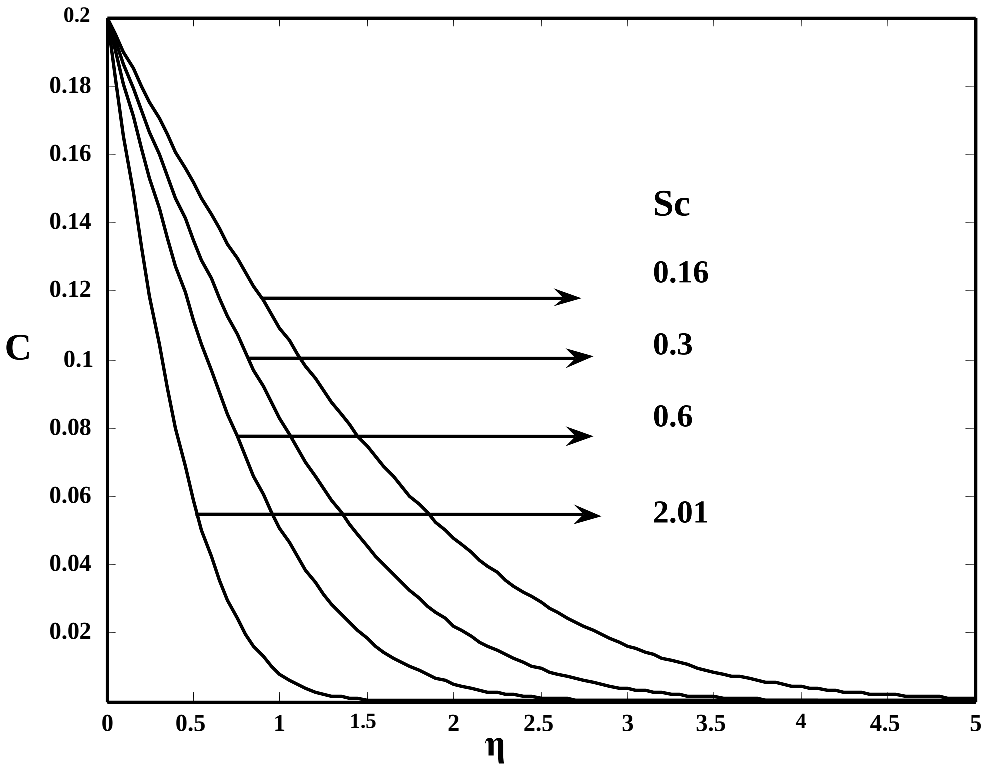

Figure 2. Concentration profiles for different Sc.

Figure 4 illustrates the velocity profiles for different Grashof numbers, (Gr = 2, 5, 10) and mass Grashof numbers (Gc = 5, 10), Ω = 0.1, Pr = 7, K = 8, on the primary velocity at t = 0.4. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number. Figure 5 demonstrates the effects of different thermal Grashof numbers (Gr =2, 5), mass Grashof numbers (Gc = 5, 10), Ω = 0.1, Pr = 7, K = 8, on the secondary velocity at time t = 0.1. It is observed that the velocity increases with decreasing values of the thermal Grashof number or mass Grashof number. The secondary velocity profiles for different rotation parameters (Ω = 0.5, 1), Gr = Gc = 10, Pr = 7, and t = 0.05 are shown in Figure 6. It is observed that the velocity decreases with increasing values of the rotation parameter Ω.

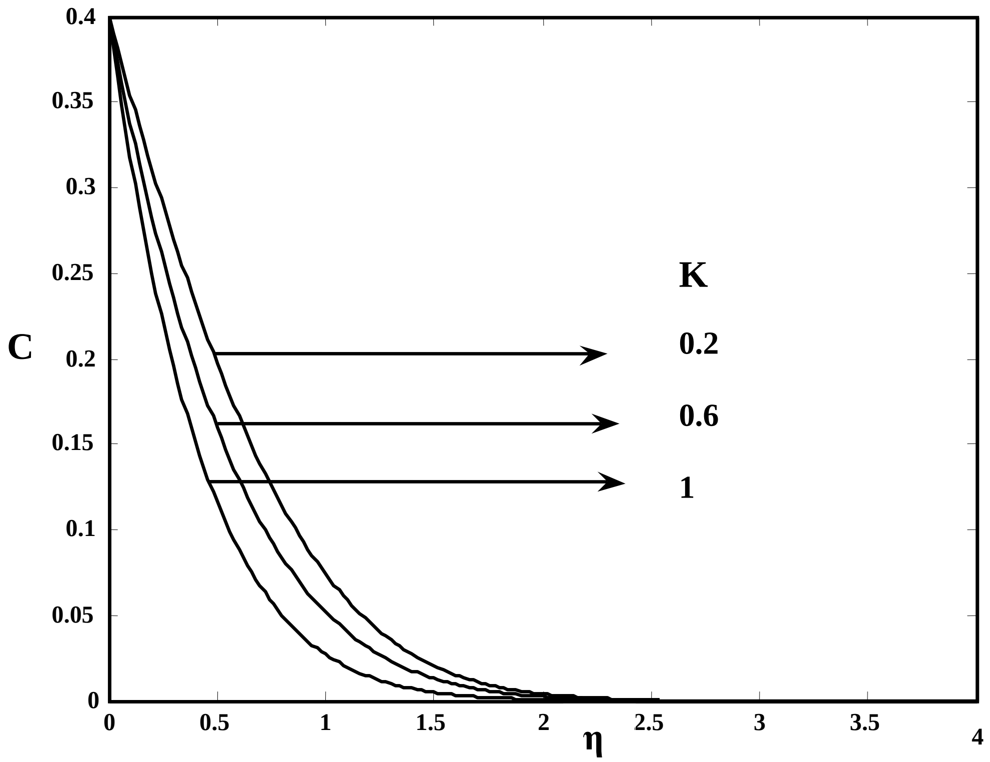

Figure 3. Concentration profiles for different K.

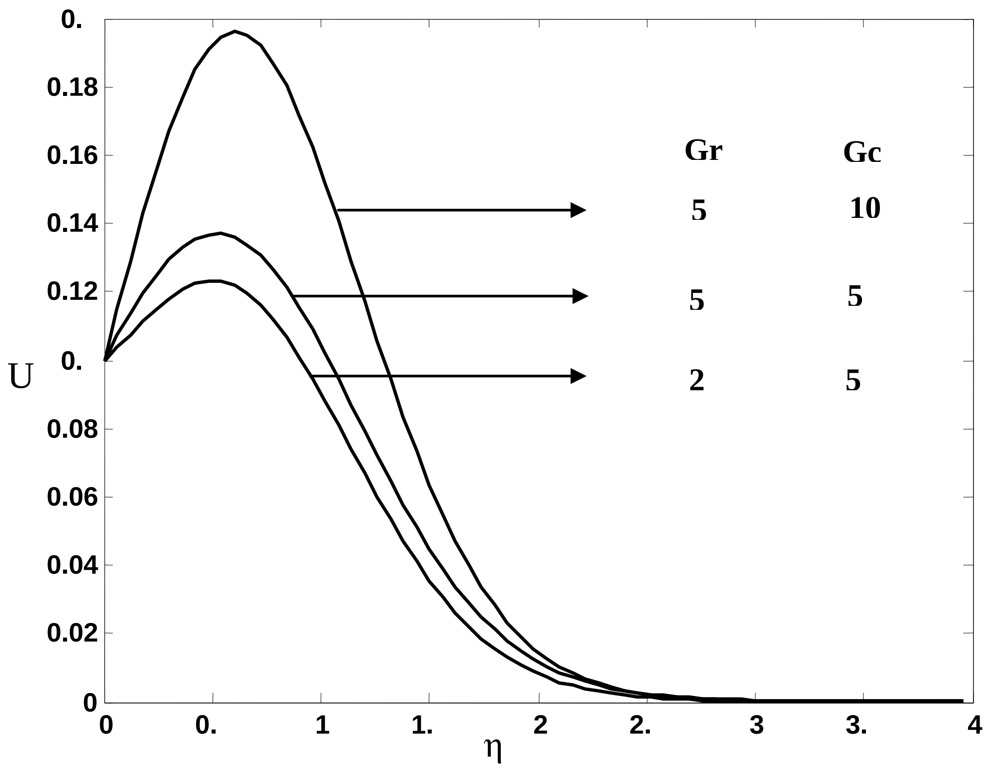

Figure 4. Primary velocity profiles for different values of Gr and Gc.

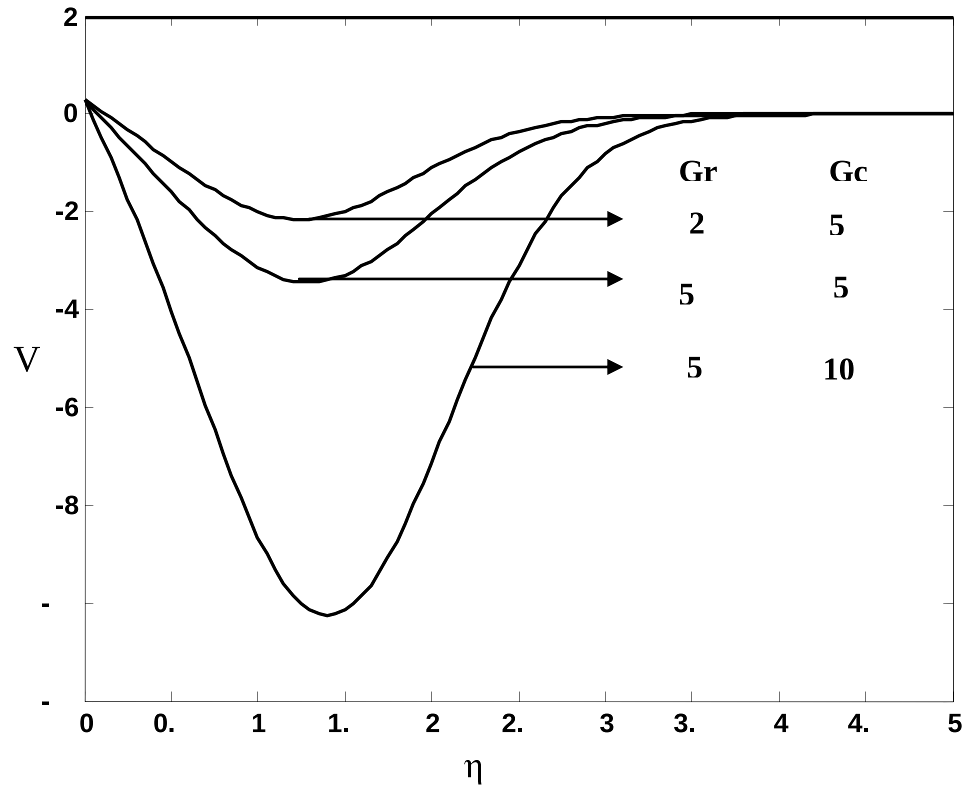

Figure 5. Secondary velocity profiles for different values of Gr and Gc.

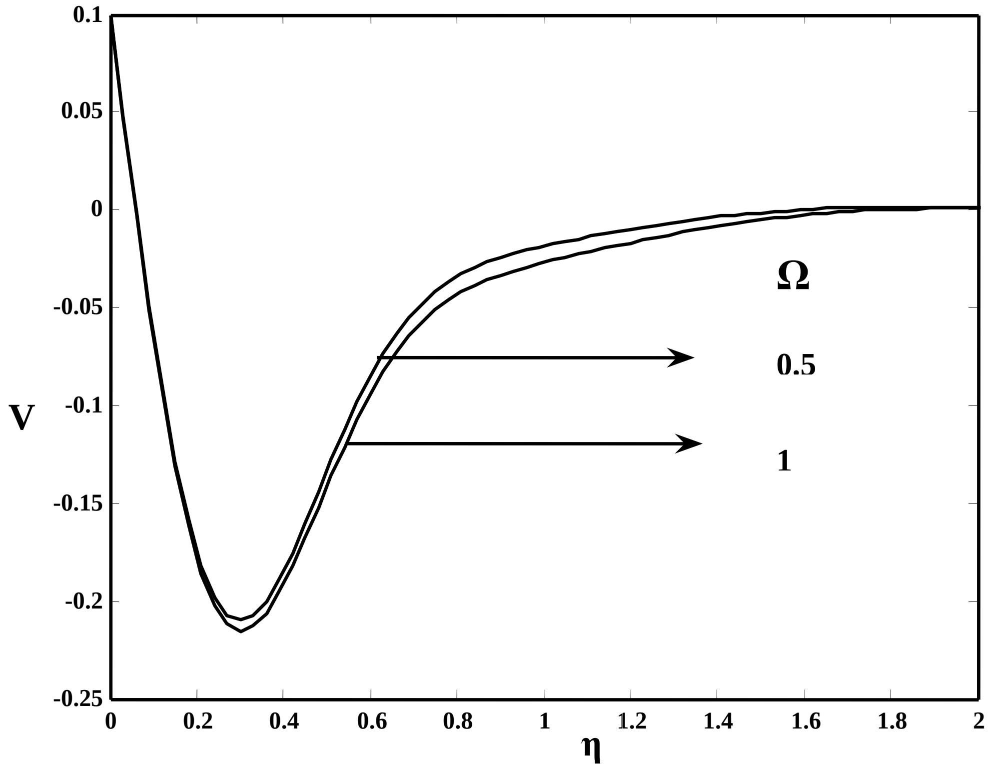

Figure 6. Secondary velocity profiles for different Ω.

CONCLUSION

The theoretical solution of flow past a uniformly accelerated infinite isothermal vertical plate in the presence of variable mass diffusion has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effects of different physical parameters, like thermal Grashof number, mass Grashof number and time, are studied graphically. It is observed that the velocity increases with increasing values of *Gr*, *Gc* and *t*. However, the trend is simply reversed with respect to the rotation parameter.

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NOMENCLATURE

A	-	constant
C'	-	species concentration in the fluid
C	-	dimensionless concentration
C_w	-	wall concentration
C_∞	-	concentration far away from the plate
C_p	-	specific heat at constant pressure
D	-	mass diffusion coefficient
Gc	-	mass Grashof number
Gr	-	thermal Grashof number
g	-	acceleration due to gravity
k	-	thermal conductivity
K	-	chemical reaction parameter
Pr	-	Prandtl number
Sc	-	Schmidt number
T	-	temperature of the fluid near the plate
T_w	-	temperature of the plate
T_∞	-	temperature of the fluid far away from the plate
t'	-	time
t	-	dimensionless time
u	-	velocity of the fluid in the x-direction
u_0	-	velocity of the plate
q	-	dimensionless velocity
x	-	spatial coordinate along the plate
y'	-	coordinate axis normal to the plate
Z	-	dimensionless coordinate axis normal to the plate
β	-	volumetric coefficient of thermal expansion
β^*	-	volumetric coefficient of expansion with concentration
Ω	-	rotation parameter
μ	-	coefficient of viscosity
υ	-	kinematic viscosity
ρ	-	density of the fluid
τ	-	dimensionless skin-friction kg.
θ	-	dimensionless temperature
η	-	similarity parameter
erfc	-	complementary error function